Eastern Mediterranean University

Department of Economics

Discussion Paper Series

Forecasting South African Inflation Using Non-Linear Models: A Weighted Loss-Based Evaluation

Pejman Bahramian , Mehmet Balcilar , Rangan Gupta, Patrick T. kanda

Discussion Paper 15-20

July 2015

Department of Economics

Eastern Mediterranean University

Famagusta, North Cyprus

Forecasting South African Inflation Using Non-linear Models: A Weighted Loss-based Evaluation

Pejman Bahramian [*] Mehmet Balcilar [†] Rangan Gupta [‡] Patrick T. Kanda [§]

April 24, 2014

Abstract

The conduct of inflation targeting is heavily dependent on accurate inflation forecasts. Non-linear models have increasingly featured, along with linear counterparts, in the forecasting literature. In this study, we focus on forecasting South African inflation by means of non-linear models and using a long historical dataset of seasonally-adjusted monthly inflation rates spanning from 1921:02 to 2013:01. For an emerging market economy such as South Africa, non-linearities can be a salient feature of such long data, hence the relevance of evaluating non-linear models' forecast performance. In the same vein, given the fact that 1969:10 marks the beginning of a protracted rising trend in South African inflation data, we estimate the models for an in-sample period of 1921:02-1966:09 and evaluate 24 step-ahead forecasts over an out-of-sample period of 1966:10-2013:01. In addition, using a weighted loss function specification, we evaluate the forecast performance of different non-linear models across various extreme economic environments and forecast horizons. In general, we find that no competing model consistently and significantly beats the LoLiMoT's performance in forecasting South African inflation.

Keywords: Inflation, forecasting, non-linear models, weighted loss function, South Africa
JEL Codes: C32, E31, E52

1 Introduction

An inflation targeting framework requires a central bank's policy actions to be informed by expected future inflation in relation to a publicized inflation target (Green, 1996). Under such a framework, monetary policy instruments are used to bring inflation forecasts close to the inflation target (Croce and Khan, 2000). In February 2000, South Africa adopted an inflation target range of between 3 to 6 per cent. As Leiderman and Svensson (1995) observe, good forecasting models as well as insights into the transmission mechanisms of monetary policy are indispensable in an inflation targeting regime. Against this backdrop, the South African Reserve Bank relies on a suite of models to forecast inflation so as to support the conduct of monetary policy (van den Heever, 2001).

[*]Department of Economics, Eastern Mediterranean University, Famagusta, Turkish Republic of Northern Cyprus, via Mersin 10, Turkey.

[†]Department of Economics, Eastern Mediterranean University, Famagusta, Turkish Republic of Northern Cyprus, via Mersin 10, Turkey.

[‡]*Corresponding author*, Department of Economics, University of Pretoria, Pretoria 0002, South Africa. Phone: +27 (0)12 420 3460, email: Rangan.Gupta@up.ac.za.

[§]Department of Economics, University of Pretoria, Pretoria 0002, South Africa.

Forecasting economic variables - including inflation - by means of non-linear models has recently gained momentum in the literature (see for example Balcilar *et al.*, 2012; Teräsvirta, 2006; Binner *et al.*, 2005; Clements *et al.*, 2003; Ascari and Marrocu, 2003; Chauvet *et al.*, 2002 and Stock and Watson, 1998). Most studies have compared the forecasting performance of non-linear models to that of linear counterparts. Ascari and Marrocu (2003) forecast US inflation using a linear (NAIRU Phillips curve) and non-linear (threshold autoregressive) models. They find that, while the non-linear model performs better in capturing the distribution of inflation, the Phillips curve's forecast performance is superior in terms of mean-squared forecast errors over the medium to long-term horizon. Stock and Watson (1998) compare the performance of autoregressions, exponential smoothing, artificial neural networks and smooth transition autoregression models in forecasting US macro variables. Their findings are that autoregression models with unit root pretests perform better than other models. The authors also observe that the forecast performance can improve when combined with other models' forecasts. On the other hand, Chauvet *et al.* (2002) compare the ability of linear and non-linear models in forecasting Brazilian GDP growth. They conclude that non-linear models outperform linear counterparts. In addition, the authors observe that allowing for structural breaks in the data provides a good representation of Brazil's business cycle. In a study investigating the performance of linear and non-linear models in forecasting Euro inflation, Binner *et al.* (2005) observe that linear models are restricted in the event non-linearities are present in the data. The authors find that the non-linear (neural networks) model generates better in- and out-of-sample forecasts of EU inflation compared to linear (*i.e.* VAR and ARIMA) models. Balcilar *et al.* (2012) evaluate the out-of-sample performance of linear and non-linear models in forecasting US and Census regions housing prices. The authors conclude that the non-linear smooth-transition autoregressive model beats the linear autoregressive model in point forecasts at longer horizons. On the other hand, the linear autoregressive model does better at shorter horizons.

There exists a vast literature on comparing the performance of different models in forecasting inflation in South Africa (see for example Balcilar *et al.*, 2013; Gupta and Hartley, 2013; Gupta and Steinbach, 2013, Kanyama and Thobejane, 2013; Aron and Muellbauer, 2012; Alpanda *et al.*, 2011; Gupta and Kabundi, 2011a, 2011b, 2010; Liu *et al.*, 2009; Woglom, 2005; and Pretorius and van Rensburg, 1996).[1] Most of these studies, except for Balcilar *et al.* (2013) and Kanyama and Thobejane (2013), rely on linear (or linearized) methods to forecast South African inflation. On the whole, findings are that different model specifications beat benchmark time-series models (*e.g.* ARIMA and Vector Autoregressive (VAR)) models in forecasting South African inflation. To illustrate, Aron and Muellbauer (2012) observe that cointegrated equations for South African relative prices outperform univariate benchmark models in forecasting inflation in South Africa. In studies using large factor models to, among others, forecast South African inflation, Gupta and Kabundi (2010, 2011a, 2011b) conclude that, given the blessings of dimensionality, factor models tend to outperform an unrestricted VAR, Minnesota-prior-based Bayesian VARs (BVARs) and typical closed and small open economy New Keynesian Dynamic Stochastic General Equilibrium (NKDSGE) models. However more recently, Liu *et al.* (2009), Alpanda *et al.* (2011) and Gupta and Steinbach (2013) develop more sophisticated (allowing for various real and nominal rigidities) closed and small open economy NKDSGE models for South Africa and conclude that it outperforms the BVAR and VAR in forecasting inflation. An early study by Pretorius and van Rensburg (1996) argue that the Phillips curve model outperforms the

[1] Note studies by Gupta and Sichei (2006) and Gupta (2006) depicted the dominance of Bayesian Vector autoregressive (BVAR) and Bayesian Vector Error Correction (BVECM) models in predicting CPI in log-levels rather than inflation over their classical counterparts.

traditional monetarist, money demand as well as time-series (ARIMA) models in forecasting South African inflation. However, in a study investigating the determinants of inflation in South Africa, Woglom (2005) posits that inflation forecasts from a simple Phillips curve are not very accurate. Gupta and Hartley (2013) highlights the role of asset prices in forecasting inflation, by pointing out that Autoregressive Distributed Lag (ARDL) models that include information on asset prices outperforms simple autoregressive (AR) models.

In a new strand of research, Balcilar *et al.* (2013) and Kanyama and Thobejane (2013) use linear and non-linear frameworks to forecast inflation in South Africa. One one hand, Balcilar *et al.* (2013) apply a non-linear NKDSGE model to South African macroeconomic data. They find that the performance of the non-linear NKDSGE model in forecasting South African inflation beats that of the linear NKDSGE model, as well as, a selection of VAR and BVAR models, both constant parameter and time-varying. This highlights the importance of including non-linearities in a DSGE framework to take into account structural changes, especially in an emerging market economy such as South Africa. Note that though, the "non-linearity" of the approach only appears in the estimation technique but not assumed in the data generating process. On the other hand, Kanyama and Thobejane (2013) compare parametric and non-parametric models in forecasting inflation in South Africa and find that the Kernel regression non-parametric model's forecast performance outperforms that of the ARIMA models.

Against this backdrop, we contribute to the literature on forecasting inflation in South Africa in three ways. First, we focus on evaluating the performance of a suite of non-linear models (namely, Locally Linear Model Tree, Multi-Layered Perceptron, Non-linear Autoregressive, Genetic Algorithm) relative to a linear benchmark model (Autoregressive), as well as, a simple forecast combination model (MEAN) in forecasting South African inflation. Second, whereas the inflation forecasting literature for South Africa primarily uses quarterly consumer prices as a measure of inflation with an out-of-sample forecasting period covering the post inflation targeting period starting in 2000, our study relies on a much longer dataset at a higher frequency, which is of paramount importance to a country targeting inflation. We use the seasonally-adjusted month-on-month percentage change in consumer prices over the period 1921:02-2013:01 (the out-of-sample forecasting period being 1966:10-2013:01). The fact that a model's performance in forecasting inflation can hinge on the sample period under consideration motivates our choice of a long historical dataset tracing South African inflation as far back as 1921:02 - the earliest possible date for which month-on-month inflation data is available for South Africa (given that the monthly consumer price index data starts in 1921:01). In addition, the beginning of an upward trend in the inflation data after 1966:09 determines our choice for the out-of-sample forecasting period, which also covers all major monetary policy changes in the South African economy. Third, contrary to the usual practice in the literature of evaluating model forecasts - which relies on a standard loss function - we follow van Dijk and Franses (2003) who propose a forecasting evaluation framework based on a weighted loss function. This method allows us to identify models that, in addition to being a good choice on average, perform better in forecasting extreme events - *e.g.* periods of high or low inflation. In this regard, we use the weighted root mean squared error statistic, as well as, the weighted variant of the Diebold-Mariano (1995) test to compare the forecasting performance of the different models.

In what follows, Section 2 discusses the methodology. Next, we present empirical results in Section 3. Finally, Section 4 concludes the paper.

2 Methodology

2.1 Forecasting models

Although complex in application, non-linear models have been shown to outperform regression models in modelling both linear and non-linear relationships via logistic functions (Bishop, 1995). Given this backdrop, we forecast inflation in South Africa using a suite of non-linear models: Multi-Layered Perceptron (MLP), Non-linear Autoregressive (NAR), Genetic Algorithm (GA) and Locally Linear Model Tree (LoLiMoT). Essentially, the models are used to do a 24-step ahead out-of-sample forecast. Table 1 presents the description of the models.

[Insert Table 1]

Stock and Watson (1999, 2003, 2004) show that blending different approaches works better in forecasting inflation and output using a large number of competing predictors. In this regard, we compute the combination forecasts (MEAN) from the four models.

To evaluate the performance of individual models as well as the forecast combination method in forecasting inflation in South Africa, we compute and rank forecast errors across different forecast horizons. To add more substance to the comparison, we also include forecast errors from a benchmark autoregressive model (AR) of order 5.[2]

2.2 Forecast evaluation using weighted loss functions

The common way in the forecast evaluation literature of assessing the forecast performance of a model is to use the standard period-t loss function defined by the squared forecast error

$$\mathcal{L}_{i,t} = e_{i,t}^2, \tag{1}$$

where $e_{i,t} = y_t - y_{i,t}^f$ represents model i's forecast error, y_t is the target variable's realization, $y_{i,t}^f$ is the value forecast by model i.

To compare the average loss difference of two contending models - *e.g.* models 1 and 2 - one needs to compute their respective mean squared forecast errors (MSFE) as follows

$$MSFE_i = \frac{1}{P} \sum_{t=T+1}^{T+P} e_{i,t}^2, \qquad i = 1, 2, \tag{2}$$

over the period $T+1$ to $T+P$ (*i.e.* the forecast period) and select the model yielding the smallest MSFE.[3]

Nonetheless, to an applied forecaster or a consumer of forecasts (*e.g.* a company chief executive officer or a politician), different situations may warrant diverse nuances in the specification of the loss function

[2] The order of the autoregressive model is chosen using the Akaike Information Criterion (AIC).
[3] $RMSE = \sqrt{MSFE}$

(Carstensen *et al.*, 2010). To illustrate, forecasting reasonably well an extreme event such as the 2008-2009 global recession could require more than merely seeking to minimize a model's average squared error.

Against this backdrop, van Dijk and Franses (2003) propose a weighted squared forecast error specification of the loss function to assign heavy weights on extreme events. The weighted loss function is given by

$$\mathcal{L}^{w}_{i,t} = w_t e^2_{i,t}, \tag{3}$$

where the weight w_t is defined as

1. $w_{left,t} = 1 - \widehat{F}(y_t)$, where $F(.)$ represents the cumulative distribution of y_t, to impose heavier weights on the distribution's left tail. Such a specification defines a "low inflation" loss function.

2. $w_{right,t} = \widehat{F}(y_t)$, to attach heavier weights on the distribution's right tail. Such a specification defines a "high inflation" loss function.

3. $w_{tail,t} = 1 - \widehat{F}(y_t)/max(\widehat{F}(y_t))$, where $F(.)$ represents the density of y_t. Such specification places heavier weights on both tails of the distribution. In this regard, we have a low and high inflation loss function.

In the event $w_t = 1$, then the weighted loss function (3) reduces to the standard "uniform" loss function.

Evaluating a forecast model i over a forecast horizon $T + 1$ to $T + P$, requires computing the weighted mean squared forecast error

$$MSFE_i = \frac{1}{P}\sum_{t=T+1}^{T+P} w_t e^2_{i,t}. \tag{4}$$

To establish the forecasting performance of model i relative to that of a benchmark model 0 requires calculating the weighted loss difference

$$d_{i,t} = \mathcal{L}^{w}_{0,t} - \mathcal{L}^{w}_{i,t} = w_t e^2_{0,t} - w_t e^2_{i,t} \tag{5}$$

and averaging over the forecast horizon

$$\bar{d}_i = \frac{1}{P}\sum_{t=T+1}^{T+P} d_{i,t} = \frac{1}{P}\sum_{t=T+1}^{T+P} w_t e^2_{0,t} - \frac{1}{P}\sum_{t=T+1}^{T+P} w_t e^2_{i,t} \tag{6}$$

We use van Dijk and Franses (2003)'s weighted loss function framework to investigate the performance of five non-linear models in forecasting inflation in South Africa.

Furthermore, to ascertain whether empirical loss differences between two contending models are statistically significant, we use a modified version of Diebold and Mariano (1995)'s pairwise test, developed by Harvey *et al.* (1997) to correct for small sample bias. The modified Diebold-Mariano (1995) - henceforth MDM - test compares the forecast accuracy of two models at a time. Essentially, the MDM test assesses whether the average loss differences between a pair of models is significantly different from zero. Given a benchmark model 0 and an alternative model i, the null hypothesis of the MDM test is that of equal forecast performance,

$$E[d_{i,t}] = E[\mathcal{L}^w_{0,t} - \mathcal{L}^w_{i,t}] = 0 \tag{7}$$

Harvey *et al.* (1997) define the MDM test statistic as

$$MDM = \left(\frac{P + 1 - 2h + P^{-1}h(h-1)}{P} \right)^{1/2} \widehat{V}(\bar{d}_i)^{-1/2} \bar{d}_i, \tag{8}$$

where h represents the forecast horizon and $\widehat{V}(\bar{d}_i)$ is the variance of $d_{i,t}$. The MDM test statistic is compared to a critical value obtained from the student's t-distribution with $P - 1$ degrees of freedom.

2.3 Data description

We measure inflation rates as the seasonally-adjusted[4] month-on-month growth rates of Consumer Price Index (CPI) data for South Africa, obtainable from the Global Financial Database. Essentially, our sample covers the period 1921:02-2013:01, *i.e.* a total of 1104 observations. Data availability at the timing of writing the paper determines the start and end dates of the sample. Given the break in inflation data around 1966:09, after which inflation starts rising (see Figure 1), we estimate the models using an in-sample period of 1921:02-1966:09 (*i.e.* 548 observations), while, the out-of-sample forecasting period is 1966:10-2013:01 (*i.e.* 556 observations). This in-sample and out-of-sample splits also gives us nearly equal number of observations, which is enough to estimate the models, especially the data-intensive non-linear ones, with high degree of precision.

[Insert Figure 1]

Figure 1 shows the historical evolution of South African monthly inflation rates (seasonally-adjusted) from 1921:02 to 2013:01. Figure 2(a) shows the inflation rates from 1966:10 to 2013:01 - our out-of-sample period. As can be seen from the figure, a rising trend in inflation starts around 1966:10 which delimits the in- and out-of-sample periods. Figure 2(b) plots the density function of inflation over the same period. And, Figures 2(c), (d) and (e) plot the weights under the high inflation periods, low inflation periods and high and low inflation periods, respectively. We note that the distribution of the inflation variable is not symmetrical. It is skewed to the right. This supports the use of different weights to penalise forecast errors under extreme periods of high or low inflation or both. It can also be seen from Figures 2(a) and (d) that low inflation weights are associated with periods where inflation falls considerably.

[Insert Figure 2]

[4]Seasonal adjustment is performed with Census X-12 method.

3 Empirical results

Based on different weighing schemes (*i.e.* uniform, high inflation period, low inflation period as well as high and low inflation periods) for the loss function, Table 2 reports the RMSE for each model [including the benchmark AR(5) model] over forecast horizons h ($h \in \{1, 4, 12, 24\}$) as well as the associated ranking.

[Insert Table 2]

Using a uniform weights scheme, the LoLiMoT model outperforms all other models across all forecast horizons as it consistently ranks first - which means it generates the smallest RMSE - in all cases. The benchmark AR model comes second when considering the very short-term forecast horizon ($h = 1$) while the MEAN model ranks second for forecasts beyond 1 month ($h = 4$, $h = 12$, and $h = 24$).

Results based on a high inflation weights scheme show that the AR model has the best forecast performance overall (except for the forecast at horizon $h = 12$ where it is outclassed by the MEAN model). The LoLiMoT model's performance under this weighting scheme is average as it consistently ranks third out of six possible places across all forecast horizons.

When considering the low inflation weights, the MLP model's forecast performance outdoes that of the other models over the very short-term horizon ($h = 1$) as well as over the long-term horizon ($h = 24$). The MEAN and LoLiMoT models rank first over the short-term ($h = 4$) and medium-term ($h = 12$) respectively. Nonetheless, given that the LoLiMoT consistently ranks among the first two best performing models across all forecast horizons and also the fact that the MLP model shows an average performance at $h = 4$, we can conclude that, on the whole, the forecasting performance of the LoLiMoT is somewhat comparable to that of the MLP model when using a recession weights scheme.

Findings based on the tail weights scheme mirror those observed under the uniform weights scheme. The LoLiMoT model outclasses all other models as it consistently yields the smallest forecasting errors across all forecast horizons. The MEAN model comes second over horizons $h = 4$, $h = 12$ and $h = 24$. Over the very short-term ($h = 1$), the AR model ranks second to the LoLiMoT model.

All in all, results show that the LoLiMoT model's performance in forecasting South African inflation is superior to that of other models under the uniform and tail weights schemes. Based on a low inflation weights scheme, the LoLiMoT model's forecast accuracy competes with that of the MLP model. However, under a high inflation weights scheme, the LoLiMoT model performs moderately in forecasting inflation in South Africa. The AR model beats all other models in forecasting South African inflation under a high inflation weights scheme, followed by the MEAN model. In contrast, the NAR and GA models have the worst forecasting performance record both across the weighting scheme used as well as the forecast horizon considered. Given this backdrop, we conclude the performance of non-linear models (as used in this study) in forecasting South African inflation appears to be largely (but not wholly) dependent on the prevailing inflationary environment. Be that as it may, unlike the other models, the LoLiMoT model consistently ranks among the top three models yielding small forecast error under different inflationary environments as well as over different forecast horizons.

Tables 3, 4, 5 and 6 report results from the MDM test which assesses the statistical significance[5] of the empirical findings on the performance of non-linear model in forecasting South Africa under different weighting schemes and across different forecast horizons.

Under a uniform weights scheme as reported in Table 3, the MDM test results show that the LoLiMoT and AR models significantly have smaller forecast errors compared to (and therefore outclass) the MLP, NAR, GA and MEAN models at $h = 1$. Although the LoLiMoT model has a smaller forecast error in comparison with that of the AR model, this, however, is shown not to be statistically significant. At $h = 4$ and $h = 12$, with the exception of the MLP and AR models - in which cases findings that the LoLiMoT models yields the smallest forecast error is not statistically significant - results show that the performance of the LoLiMoT model in forecasting South African inflation significantly outclasses that of other models (*i.e.* NAR, GA and MEAN). Similarly, at $h = 24$, we find that the LoLiMoT model is, on balance, the best performer. It significantly has a smaller forecast error relative to that of the AR, NAR and GA models. Furthermore, no model significantly beats the performance of the LoLiMoT model in forecasting longer-term South African inflation using a uniform weights scheme.

[Insert Table 3]

Table 4 reports MDM test findings based on a "high inflation" weights scheme. At $h = 1$, results show that no model outperforms the LoLiMoT and AR models in forecasting South African inflation. In addition, while the LoLiMoT model while the LoLiMoT model significantly beats the forecasting capability of three out of five models (*i.e.* MLP, NAR and GA), the performance of the AR benchmark significantly outdoes that of four out of five models (*i.e.* MLP, NAR, GA and MEAN). In this regard, the AR model perform marginally better than the LoLiMoT model. The outcome at $h = 1$ is comparable to that at $h = 4$ and $h = 12$. when considering both forecast horizons ($h = 4$ and $h = 12$), neither the LoLiMoT model nor the AR benchmark is outperformed by any other model. However, in each case, the AR model beats three out of five models (*i.e.* NAR, GA and MEAN) in forecasting South African inflation; whereas the LoLiMoT model only significantly outperforms two out of five models (*i.e.* NAR, and GA). At $h = 24$, each of the LoLiMoT, AR and MEAN models is not outperformed by any other other model but outclasses the GA and NAR models. The MLP model neither outperforms nor is outperformed by any model.

[Insert Table 4]

As shown in Table 5 - which reports results of the MDM test under a "low inflation" weights scheme, LoLiMoT and MLP are the only models that are not consistently and significantly outperformed by any other model across different forecast horizons. Results at $h = 1$ and $h = 4$ point to a tie between the LoLiMoT and MLP models - both significantly outperform a set of three models (*i.e.* NAR, GA and MEAN). Beyond that, the forecast performance of the two models decouples. At $h = 12$ the MLP model significantly beats four out of five competing models (*i.e.* AR, MLP, NAR and GA) while the LoLiMoT only significantly outperforms two out of five contending models (*i.e.* NAR and GA). At $h = 24$ the LoLiMoT

[5]We use a 10 per cent level of significance throughout.

model significantly outdoes two out of five competing models (*i.e.* AR and GA) whereas the MLP model significantly outperforms none of the models.

[Insert Table 5]

Table 6 presents the MDM test findings based on a tail weights scheme. The LoLiMoT model is consistently and significantly not outperformed by any of the other competing models across all forecast horizons. The LoLiMoT model's performance in forecasting South African inflation beats significantly that of four out of five contending models at horizons $h = 1$ (*i.e.* MLP, NAR, GA and MEAN) , $h = 12$ (*i.e.* AR, NAR, GA and MEAN) and $h = 24$ (*i.e.* AR, MLP, NAR and GA). Even better, the LoLiMoT model significantly outperforms all other competing models at $h = 4$.

[Insert Table 6]

4 Conclusion

This paper evaluates the performance of a suite of non-linear models in forecasting South African inflation. Using different weighting schemes to establish the forecasting performance of the models under different economic conditions related to inflation levels, we find that, no competing model beats the LoLiMoT model's performance in forecasting South African inflation. This is irrespective of weighing schemes as well as forecast horizons under consideration. On the whole, we observe that the LoLiMoT model performs consistently and significantly better than other models in forecasting South African inflation. The performance of other models is mixed across different schemes and forecast horizons. However, we find the NAR model to be the worst performer as it does not significantly outperform any of the other competing model.

Our findings, coupled with the increasing interest in using non-linear models in empirical economics as well as the fact that non-linearities can be present in macroeconomic series, show that non-linear methods should also feature prominently in the suite of forecasting models used by an inflation targeting central bank such as the South African Reserve Bank. Also, we show that, unlike the common practice of evaluating forecast evaluation using a standard loss function, different weights can be assigned to forecasts errors depending on the type of extreme economic environment on which policy-makers would like to get insights.

References

[1] Alpanda S, Kotze K and Woglom G (2011) Forecasting Performance of an Estimated DSGE Model for the South African Economy, *South African Journal of Economics*, 79(1): 50-67.

[2] Aron J and Muellbauer (2012) Improving Forecasting in an Emerging Economy, South Africa: Changing Trends, Long-run Restrictions and Disaggregation, *International Journal of Forecasting*, 28: 456-476.

[3] Ascari G and Marrocu E (2003) Forecasting Inflation: A Comparison of Linear Phillips Curve Models and Nonlinear Time Series Models, Centro Ricerche Economiche Nord Sud Working Paper No. 2003-7.

[4] Balcilar M, Gupta R and Kotzé K (2013) Forecasting South African Macroeconomic Data with a Non-linear DSGE Model, University of Pretoria Department of Economics Working Paper No. 2013-13.

[5] Balcilar M, Gupta R and Miller S M (2012) The Out-of-sample Forecasting Performance of Non-linear Models of Regional Housing Prices in the US, University of Pretoria Department of Economics Working Paper No. 2012-26.

[6] Binner J M, Bissoondeeal R K, Elger T, Gazely A M and Mullineux A W (2005) A Comparison of Linear Forecasting Models and Neural Networks: Application to Euro Inflation and Euro Divisia, *Applied Economics*, 37(6): 665-680.

[7] Bishop, C M (1995) Neural Networks for Pattern Recognition, New York, Oxford University Press Inc., NY, USA.

[8] Carstensen K, Wohlrabe K and Ziegler C (2010) Predictive Ability of Business Cycle Indicators under Test: A Case Study for the Euro Area Industrial Production, University of Munich Discussion Paper No. 2010-16.

[9] Chauvet M, Lima E C R and Vasquez B (2002) Forecasting Brazilian Output in the Presence of Breaks: A Comparison of Linear and Nonlinear Models, Federal Reserve Bank of Atlanta Working Paper No. 2002-28.

[10] Clements M P, Frances P H and Swanso N R (2003) Forecasting Economic and Financial Time-series with Non-linear Models, Rutgers University Departmental Working Papepr No. 200309.

[11] Croce E and Khan M S (2000) Monetary Regimes and Inflation Targeting, *IMF Finance and Development*, 37(3).

[12] Diebold F X and Mariano R S (1995) Comparing Predictive Accuracy, *Journal of Business and Economic Statistics*, 13(3): 253-263.

[13] Green J H (1996) Inflation Targeting: Theory snd Policy Implications, *IMF Staff Papers*, 43(4): 779-795.

[14] Gupta R and Hartley F (2013) The Role of Asset Prices in Forecasting Inflation and Output in South Africa, *Journal of Emerging Market Finance*, 12(3): 239-291.

[15] Gupta R and Steinbach R (2013) Forecasting Key Macroeconomic Variables of the South African Economy: A Small Open Economy New Keynesian DSGE-VAR Model, textitEconomic Modelling, 33(1): 19-33.

[16] Gupta R and Kabundi A (2011a) A Large Factor Model for Forecasting Macroeconomic Variables in South Africa, *International Journal of Forecasting*, 27: 1076-1088.

[17] Gupta R and Kabundi A (2011b) Forecasting Macroeconomic Variables Using Large Datasets: Dynamic Factor Model versus Large-Scale BVARs, *Indian Economic Review*, XXXXVI(1): 23-40.

[18] Gupta R and Kabundi A (2010) Forecasting Macroeconomic Variables in a Small Open Economy: A Comparison between Small- and Large-Scale Models, *Journal of Forecasting*, 29(1-2, Special Issue: Advances in Business Cycle Analysis and Forecasting): 168-185.

[19] Gupta R (2006) Forecasting the South African Economy with VARs and VECMs, *South African Journal of Economics*, 74(4): 611-628.

[20] Gupta R and Sichei M (2006) A BVAR Model of the South African Economy, *South African Journal of Economics*, 74(3): 391-409.

[21] Harvey D I, Leybourne S and Newbold P (1997) Testing the Equality of Prediction Mean Squared Errors, *International Journal of Forecasting*, 13: 281291.

[22] Kanyama I K and Thobejane B M (2013) Forecasting Macroeconomic Variables in South Africa: Parametric vs. Non-parametric Methods, Paper presented at the Economic Society of South Africa 2013 Conference.

[23] Leiderman L, Svensson L E O, eds. (1995) Inflation Targets, London: Centre for Economic Policy.

[24] Liu D, Gupta R and Schaling E (2009) A New-Keynesian DSGE Model for Forecasting the South African Economy, *Journal of Forecasting*, 28(5): 387-404.

[25] Pretorius C J and Janse van Rensburg T N (1996) The Forecast Performance of Alternative Models of Inflation, South African Reserve Bank Occasional Paper No. 10.

[26] Stock J W and Watson M W (2004) Combination Forecasts of Output Growth in a Seven Country Data Set, *Journal of Forecasting*, 23: 405-430.

[27] Stock J W and Watson M W (2003) Forecasting Output Growth and Inflation: The Role of Asset Prices, *Journal of Economic Literature*, 41: 788829.

[28] Stock J W and Watson M W (1999) Forecasting Inflation, *Journal of Monetary Economics*, 44: 293335.

[29] Stock J W and Watson M W (1998) A Comparison of Linear and Non-linear Univariate Models for Forecasting Macroeconomic Time-series, National Bureau for Economic Research Working Paper No. 6607.

[30] Taskaya-Temizel T and Casey M C (2005) A Comparative Study of Autoregressive Neural Network Hybrids, *Neural Networks*, 18(5-6): 781-789.

[31] Teräsvirta T (2006) Forecasting Economic Variables with Non-linear Models, *Handbook of Economic Forecasting*, 1: 413-457.

[32] van den Heever J P (2001) A Note on Inflation Targeting in South Africa, *The South African Journal of Economics*, 69(1): 168-177.

[33] van Dijk D and Franses P H (2003) Selecting a Non-linear Time-series Model Using Weighted Tests of Equal Forecast Accuracy, *Oxford Bulletin of Economics and Statistics*, 65: 727-744.

[34] Woglom G (2005) Forecasting South African Inflation, *South African Journal of Economics*, 73(2): 302-320.

Appendix

Table 1: Model description and specification

Code	Description and specification
MLP	The multi-layered perceptron (MLP) is a feed-forward network with hyperbolic-tangent (tansig) activation function for the hidden layers and a linear activation function for the output layer. The MLP can be represented as $\hat{y}_t = f(x_t) = w_2 g(w_2 x_t + w_b) = w_2 g(z_t)$, where $\hat{y}_t$ is the forecast of the inflation rate at time t, $x_t = [y_{t-1}, y_{t-2}, \ldots, y_{t-p}]'$ is the vector of p lags of the inflation, $z_t = w_2 x_t + w_b$; and w_1, w_2, w_b are the weight matrices of the hidden layer, output layer and the weight vector that connects the bias and the hidden layer. The nonlinear operator (g) represents the hidden layer with p sigmoid neurons, with input vector z_t (activation) and output vector (v_t): $v_t = g(z_t) = [g(z_{t-1}), g(z_{t-2}), \ldots, g(z_{t-p})]'$. We use the approach in Taskaya-Temizel and Casey (2005) and set the number of delays (nodes) to 5 for the input layer. In the MLP design, we use 1 hidden layer with 10 sigmoid neurons and 1 output layer. The Levenberg-Marquardt method is used for training function.
LoLiMoT	The locally linear tree model (LoLiMoT) uses five lags models of the inflation series in input-output relation. Given the input vector $x_t = [y_{t-1}, y_{t-2}, \ldots, y_{t-p}]'$, the overall prediction equation for LoLiMoT (includes linear and nonlinear part) is given by $\hat{y}_{i,t} = \omega_{i0} + \omega_{i1} y_{t-1} + \ldots + \omega_{ip} y_{t-p}$; $\hat{y}_t = \sum_{i=1}^{M} \hat{y}_{i,t} \phi_i(x_t)$, where ω_{ij} denotes the weights of the i^{th} neuron. The smoothing factor that we used is equal to 1/3. The membership function ϕ_i is Gaussian and maximum neuron is $M = 10$.
NAR	A non-linear autoregressive (NAR) model of order p is given by the following input-output relation $\hat{y}_t = f[\hat{y}_{t-1}, \ldots, \hat{y}_{t-p}, d_{t-1}, \ldots, d_{t-p}]$, where d are the targets for inflation and $\hat{y}$ the predicted values by the model, p is the output order. f represents a nonlinear function. Given the network's output at time t, the forecast is $\hat{y}_t$. Since we have the targets d_t, we can compute the error e_t as the difference between d_t and y_t. We use 5 lags as feedback delays. We also use 1 hidden layer with 10 neurons. Data is introduced recursively in the loop to estimate 24 steps ahead forecasts.
GA	Genetic Algorithm (GA) is defined as a feature of human brain and used in forecasting processes. Our implementation approximates a general function by minimizing the mean square error between the target inflation rate and lagged inflation rates as inputs. The GA design uses H chromosomes $g_{h,t} \in H$, that are binary strings divided into N genes $g^n_{h,t}$ with each of them encoding a candidate parameter $\theta^n_{h,t}$ for the argument θ^n when a chromosome $h \in \{1, \ldots, H\}$ is at time $t \in \{1, \ldots, T\}$. It can be determined as $g_{h,t} = \{g^1_{h,t}, \ldots, g^N_{h,t}\}$. As a result, each gene $n \in \{1, \ldots, N\}$ has its length equal to an integer L_n and is a string of binary entries: $g^n_{h,t} = \{g^{n,l}_{h,t}, \ldots, g^{n,Ln}_{h,t}$, $g^{n,l}_{h,t} \in \{0,1\} \forall j \in \{1, \ldots, L_n\}$. We use 5 lags and set the maximum number of iterations to 100. We also use the "Roulette-wheel" for procreation of parents.

Figure 1: Monthly inflation rates (seasonally-adjusted), South Africa, 1921:02-2013:01

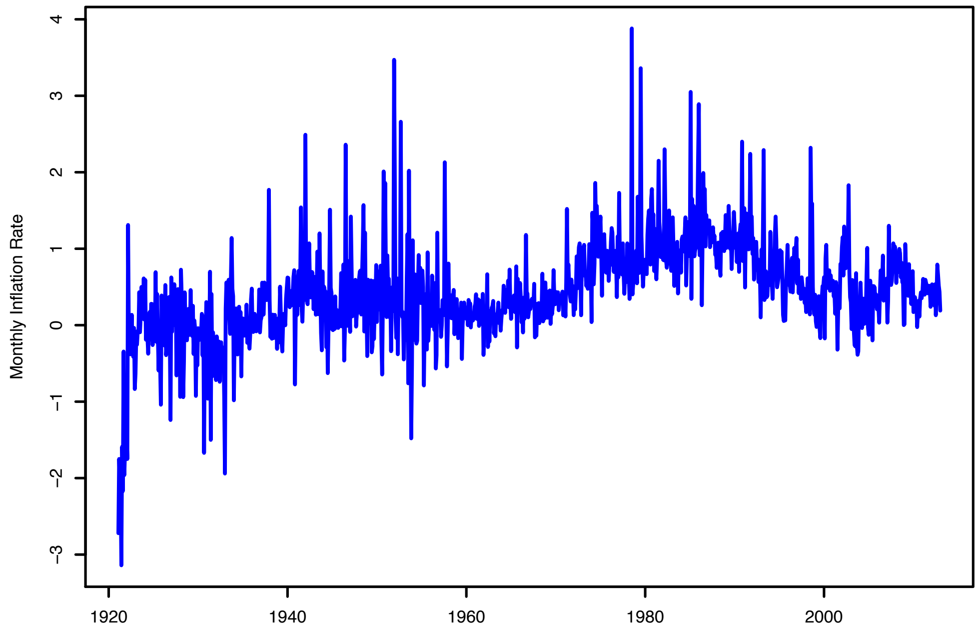

Figure 2: Inflation series, density of inflation and weights

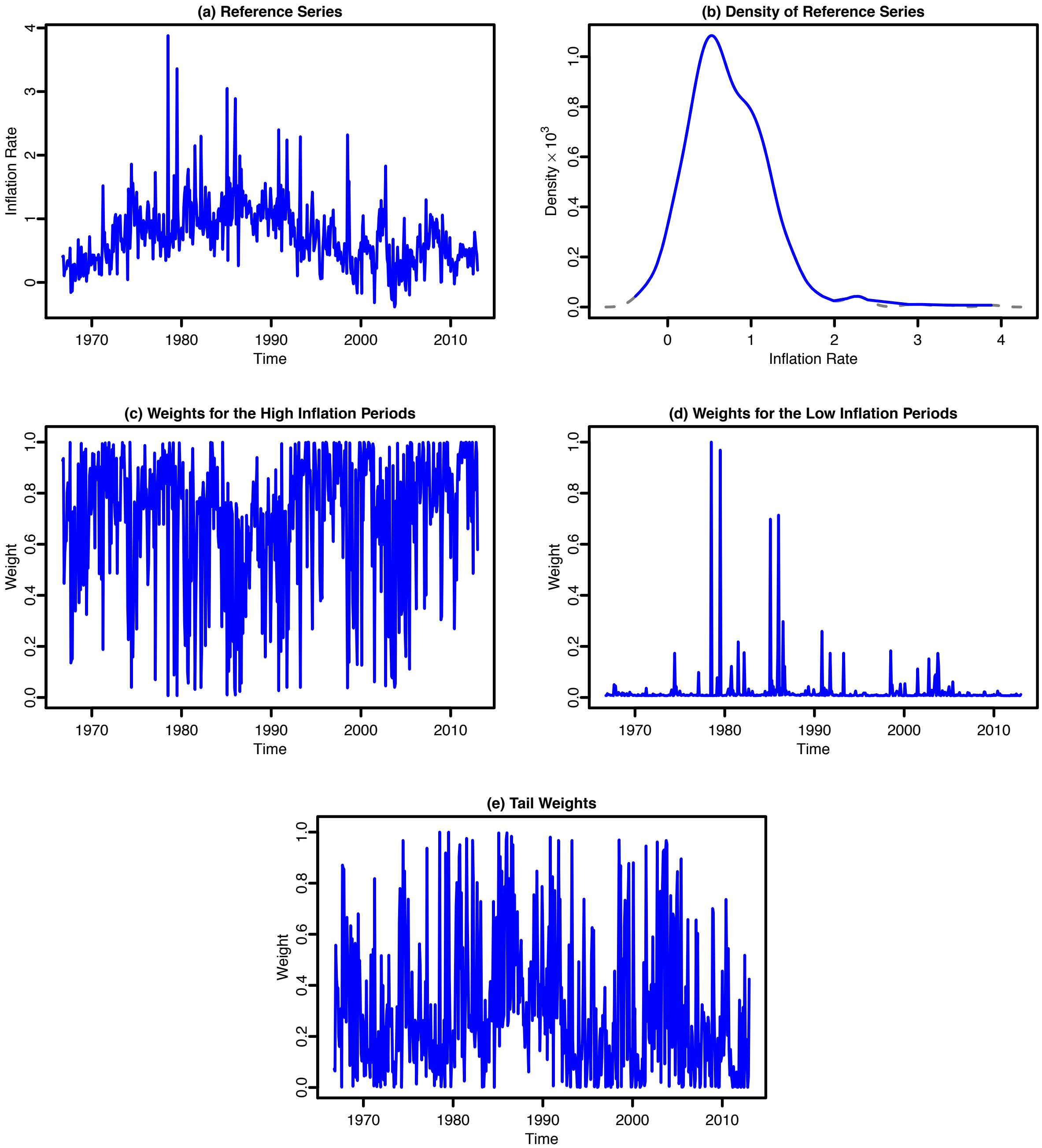

Table 2: Root Mean Squared Errors, Ranks and Weights

	Uniform weights		Boom weights		Recession weights		Tail weights	
	RMSE	Rank	RMSE	Rank	RMSE	Rank	RMSE	Rank
h=1								
LoLiMoT	0.44	1	0.21	3	0.20	2	0.33	1
AR	0.44	2	0.20	1	0.20	3	0.34	2
MLP	0.49	4	0.26	4	0.19	1	0.34	3
NAR	1.02	6	0.72	6	0.25	6	0.53	6
GA	0.57	5	0.30	5	0.23	5	0.40	5
MEAN	0.47	3	0.21	2	0.21	4	0.36	4
h=4								
LoLiMoT	0.47	1	0.22	3	0.20	2	0.35	1
AR	0.49	3	0.21	1	0.21	4	0.38	4
MLP	0.63	4	0.38	5	0.21	3	0.38	3
NAR	0.91	6	0.52	6	0.45	6	0.62	6
GA	0.65	5	0.34	4	0.23	5	0.44	5
MEAN	0.48	2	0.22	2	0.17	1	0.36	2
h=12								
LoLiMoT	0.53	1	0.27	3	0.20	1	0.38	1
AR	0.58	3	0.26	2	0.23	4	0.44	3
MLP	0.69	4	0.37	4	0.21	2	0.47	4
NAR	0.97	6	0.52	6	0.23	5	0.68	6
GA	0.82	5	0.43	5	0.24	6	0.54	5
MEAN	0.56	2	0.25	1	0.22	3	0.42	2
h=24								
LoLiMoT	0.54	1	0.28	3	0.21	2	0.39	1
AR	0.59	3	0.26	1	0.23	5	0.44	4
MLP	0.71	4	0.42	4	0.21	1	0.42	3
NAR	1.01	6	0.73	6	0.24	6	0.51	6
GA	0.88	5	0.59	5	0.23	4	0.51	5
MEAN	0.56	2	0.27	2	0.22	3	0.42	2

Note: This table reports the root mean squared errors (RMSE) as well as the corresponding ranking associated with each forecasting horizon and weighting scheme.

Table 3: Modified Diebold-Mariano test for uniform weights

h=1	LoLiMoT	AR	MLP	NAR	GA	MEAN	+	−
LoLiMoT		0.21	2.46	2.60	6.43	3.05	4	0
		(0.84)	(0.01)	(0.01)	(<0.01)	(<0.01)		
AR	-0.21		2.42	2.61	7.19	3.71	4	0
	(0.84)		(0.02)	(0.01)	(<0.01)	(<0.01)		
MLP	-2.46	-2.42		2.44	3.68	-1.37	2	2
	(0.01)	(0.02)		(0.01)	(<0.01)	(0.17)		
NAR	-2.60	-2.61	-2.44		-2.18	-2.56	0	5
	(0.01)	(0.01)	(0.01)		(0.03)	(0.01)		
GA	-6.43	-7.19	-3.68	2.18		-6.18	1	4
	(<0.01)	(<0.01)	(<0.01)	(0.03)		(<0.01)		
MEAN	-3.05	-3.71	1.37	2.56	6.18		2	2
	(<0.01)	(<0.01)	(0.17)	(0.01)	(<0.01)			

h=4	LoLiMoT	AR	MLP	NAR	GA	MEAN	+	−
LoLiMoT		0.39	1.48	2.52	5.93	2.78	3	0
		(0.70)	(0.14)	(0.01)	(<0.01)	(0.01)		
AR	-0.39		1.44	2.52	6.53	2.97	3	0
	(0.70)		(0.15)	(0.01)	(<0.01)	(<0.01)		
MLP	-1.48	-1.44		1.87	-0.46	-1.33	1	0
	(0.14)	(0.15)		(0.06)	(0.65)	(0.19)		
NAR	-2.52	-2.52	-1.87		-2.06	-2.48	0	5
	(0.01)	(0.01)	(0.06)		(0.04)	(0.01)		
GA	-5.93	-6.53	0.46	2.06		-5.87	1	3
	(<0.01)	(<0.01)	(0.65)	(0.04)		(<0.01)		
MEAN	-2.78	-2.97	1.33	2.48	5.87		2	2
	(0.01)	(<0.01)	(0.19)	(0.01)	(<0.01)			

h=12	LoLiMoT	AR	MLP	NAR	GA	MEAN	+	−
LoLiMoT		0.77	1.15	2.23	5.98	2.38	3	0
		(0.44)	(0.25)	(0.03)	(<0.01)	(0.02)		
AR	-0.77		1.11	2.22	6.44	2.41	3	0
	(0.44)		(0.27)	(0.03)	(<0.01)	(0.02)		
MLP	-1.15	-1.11		1.61	0.27	-0.99	0	0
	(0.25)	(0.27)		(0.11)	(0.79)	(0.32)		
NAR	-2.23	-2.22	-1.61		-1.67	-2.19	0	4
	(0.03)	(0.03)	(0.11)		(0.10)	(0.03)		
GA	-5.98	-6.44	-0.27	1.67		-6.08	1	3
	(<0.01)	(<0.01)	(0.79)	(0.10)		(<0.01)		
MEAN	-2.38	-2.41	0.99	2.19	6.08		2	2
	(0.02)	(0.02)	(0.32)	(0.03)	(<0.01)			

h=24	LoLiMoT	AR	MLP	NAR	GA	MEAN	+	−
LoLiMoT		2.26	1.57	3.46	6.33	0.50	3	0
		(0.02)	(0.12)	(<0.01)	(<0.01)	(0.62)		
AR	-2.26		1.38	3.37	6.71	-0.73	2	1
	(0.02)		(0.17)	(<0.01)	(<0.01)	(0.47)		
MLP	-1.57	-1.38		2.00	0.16	-1.50	1	0
	(0.12)	(0.17)		(0.05)	(0.87)	(0.13)		
NAR	-3.46	-3.37	-2.00		-2.34	-3.16	0	5
	(<0.01)	(<0.01)	(0.05)		(0.02)	(<0.01)		
GA	-6.33	-6.71	-0.16	2.34		-5.99	1	3
	(<0.01)	(<0.01)	(0.87)	(0.02)		(<0.01)		
MEAN	-0.50	0.73	1.50	3.16	5.99		2	0
	(0.62)	(0.47)	(0.13)	(<0.01)	(<0.01)			

Notes: This table reports the modifield Diebold-Mariano test statistic for each given pair of models. Numbers ( ) represent the corresponding two-sided p-value. A positive sign indicates that the forecast error of row model is smaller than that of the column model and vice versa. The last two columns count the number of times the row model significantly outperforms its competitors (column "+") and is outperformed by its competitors (column "−").

Table 4: Modified Diebold-Mariano test for boom weights

h=1	LoLiMoT	AR	MLP	NAR	GA	MEAN	+	−
LoLiMoT		-1.49	2.75	2.11	5.94	0.87	3	0
		(0.14)	(0.01)	(0.04)	(<0.01)	(0.39)		
AR	1.49		3.19	2.12	6.56	2.03	4	0
	(0.14)		(<0.01)	(0.03)	(<0.01)	(0.04)		
MLP	-2.75	-3.19		1.98	2.31	-2.58	2	3
	(0.01)	(<0.01)		(0.05)	(0.02)	(0.01)		
NAR	-2.11	-2.12	-1.98		-1.87	-2.12	0	5
	(0.04)	(0.03)	(0.05)		(0.06)	(0.03)		
GA	-5.94	-6.56	-2.31	1.87		-5.43	1	4
	(<0.01)	(<0.01)	(0.02)	(0.06)		(<0.01)		
MEAN	-0.87	-2.03	2.58	2.12	5.43		3	1
	(0.39)	(0.04)	(0.01)	(0.03)	(<0.01)			
h=4	LoLiMoT	AR	MLP	NAR	GA	MEAN	+	−
LoLiMoT		-1.59	1.36	1.96	6.76	0.79	2	0
		(0.11)	(0.18)	(0.05)	(<0.01)	(0.43)		
AR	1.59		1.38	1.98	7.66	2.01	3	0
	(0.11)		(0.17)	(0.05)	(<0.01)	(0.04)		
MLP	-1.36	-1.38		1.27	-0.83	-1.34	0	0
	(0.18)	(0.17)		(0.20)	(0.41)	(0.18)		
NAR	-1.96	-1.98	-1.27		-1.70	-1.97	0	4
	(0.05)	(0.05)	(0.20)		(0.09)	(0.05)		
GA	-6.76	-7.66	0.83	1.70		-6.74	1	3
	(<0.01)	(<0.01)	(0.41)	(0.09)		(<0.01)		
MEAN	-0.79	-2.01	1.34	1.97	6.74		2	1
	(0.43)	(0.04)	(0.18)	(0.05)	(<0.01)			
h=12	LoLiMoT	AR	MLP	NAR	GA	MEAN	+	−
LoLiMoT		-1.02	1.17	1.80	7.15	1.09	2	0
		(0.31)	(0.24)	(0.07)	(<0.01)	(0.27)		
AR	1.02		1.21	1.81	7.78	1.91	3	0
	(0.31)		(0.22)	(0.07)	(<0.01)	(0.06)		
MLP	-1.17	-1.21		1.24	-0.44	-1.15	0	0
	(0.24)	(0.22)		(0.21)	(0.66)	(0.25)		
NAR	-1.80	-1.81	-1.24		-1.51	-1.79	0	3
	(0.07)	(0.07)	(0.21)		(0.13)	(0.07)		
GA	-7.15	-7.78	0.44	1.51		-6.23	0	3
	(<0.01)	(<0.01)	(0.66)	(0.13)		(<0.01)		
MEAN	-1.09	-1.91	1.15	1.79	6.23		2	1
	(0.27)	(0.06)	(0.25)	(0.07)	(<0.01)			
h=24	LoLiMoT	AR	MLP	NAR	GA	MEAN	+	−
LoLiMoT		-0.06	1.51	5.69	6.98	0.68	2	0
		(0.95)	(0.13)	(<0.01)	(<0.01)	(0.50)		
AR	0.06		1.51	5.69	7.63	1.02	2	0
	(0.95)		(0.13)	(<0.01)	(<0.01)	(0.31)		
MLP	-1.51	-1.51		1.37	-0.40	-1.50	0	0
	(0.13)	(0.13)		(0.17)	(0.69)	(0.13)		
NAR	-5.69	-5.69	-1.37		-3.53	-5.79	0	4
	(<0.01)	(<0.01)	(0.17)		(<0.01)	(<0.01)		
GA	-6.98	-7.63	0.40	3.53		-8.01	1	3
	(<0.01)	(<0.01)	(0.69)	(<0.01)		(<0.01)		
MEAN	-0.68	-1.02	1.50	5.79	8.01		2	0
	(0.50)	(0.31)	(0.13)	(<0.01)	(<0.01)			

Note: see notes to Table 2.

Table 5: Modified Diebold-Mariano test for recession weights

h=1	LoLiMoT	AR	MLP	NAR	GA	MEAN	+	−
LoLiMoT		0.52	-0.78	3.04	2.56	2.44	3	0
		(0.61)	(0.43)	(<0.01)	(0.01)	(0.01)		
AR	-0.52		-1.38	2.76	2.08	1.21	2	0
	(0.61)		(0.17)	(0.01)	(0.04)	(0.23)		
MLP	0.78	1.38		2.77	2.29	1.99	3	0
	(0.43)	(0.17)		(0.01)	(0.02)	(0.05)		
NAR	-3.04	-2.76	-2.77		-2.94	-2.99	0	5
	(<0.01)	(0.01)	(0.01)		(<0.01)	(<0.01)		
GA	-2.56	-2.08	-2.29	2.94		-2.39	1	4
	(0.01)	(0.04)	(0.02)	(<0.01)		(0.02)		
MEAN	-2.44	-1.21	-1.99	2.99	2.39		2	2
	(0.01)	(0.23)	(0.05)	(<0.01)	(0.02)			

h=4	LoLiMoT	AR	MLP	NAR	GA	MEAN	+	−
LoLiMoT		1.27	1.54	1.58	2.46	1.58	1	0
		(0.21)	(0.12)	(0.11)	(0.01)	(0.11)		
AR	-1.27		0.60	1.53	2.41	1.05	1	0
	(0.21)		(0.55)	(0.13)	(0.02)	(0.29)		
MLP	-1.54	-0.60		1.22	1.91	-0.28	1	0
	(0.12)	(0.55)		(0.22)	(0.06)	(0.78)		
NAR	-1.58	-1.53	-1.22		-0.67	-1.48	0	0
	(0.11)	(0.13)	(0.22)		(0.50)	(0.14)		
GA	-2.46	-2.41	-1.91	0.67		-2.49	0	4
	(0.01)	(0.02)	(0.06)	(0.50)		(0.01)		
MEAN	-1.58	-1.05	0.28	1.48	2.49		1	0
	(0.11)	(0.29)	(0.78)	(0.14)	(0.01)			

h=12	LoLiMoT	AR	MLP	NAR	GA	MEAN	+	−
LoLiMoT		0.90	-1.32	2.71	1.98	1.51	2	0
		(0.37)	(0.19)	(0.01)	(0.05)	(0.13)		
AR	-0.90		-1.69	2.75	2.19	1.74	3	1
	(0.37)		(0.09)	(0.01)	(0.03)	(0.08)		
MLP	1.32	1.69		2.57	2.09	1.87	4	0
	(0.19)	(0.09)		(0.01)	(0.04)	(0.06)		
NAR	-2.71	-2.75	-2.57		0.26	-2.89	0	4
	(0.01)	(0.01)	(0.01)		(0.80)	(<0.01)		
GA	-1.98	-2.19	-2.09	-0.26		-2.11	0	4
	(0.05)	(0.03)	(0.04)	(0.80)		(0.04)		
MEAN	-1.51	-1.74	-1.87	2.89	2.11		2	2
	(0.13)	(0.08)	(0.06)	(<0.01)	(0.04)			

h=24	LoLiMoT	AR	MLP	NAR	GA	MEAN	+	−
LoLiMoT		2.13	1.11	1.04	2.15	-0.77	2	0
		(0.03)	(0.27)	(0.30)	(0.03)	(0.44)		
AR	-2.13		-0.33	1.02	1.72	-1.03	1	1
	(0.03)		(0.74)	(0.31)	(0.09)	(0.30)		
MLP	-1.11	0.33		1.03	1.58	-0.89	0	0
	(0.27)	(0.74)		(0.30)	(0.11)	(0.37)		
NAR	-1.04	-1.02	-1.03		-0.96	-1.02	0	0
	(0.30)	(0.31)	(0.30)		(0.34)	(0.31)		
GA	-2.15	-1.72	-1.58	0.96		-1.42	0	2
	(0.03)	(0.09)	(0.11)	(0.34)		(0.16)		
MEAN	0.77	1.03	0.89	1.02	1.42		0	0
	(0.44)	(0.30)	(0.37)	(0.31)	(0.16)			

Note: see notes to Table 2.

Table 6: Modified Diebold-Mariano test for tail weights

h=1	LoLiMoT	AR	MLP	NAR	GA	MEAN	+	−
LoLiMoT		1.24	1.72	4.50	4.98	4.29	4	0
		(0.21)	(0.09)	(<0.01)	(<0.01)	(<0.01)		
AR	-1.24		1.09	4.50	5.04	3.59	3	0
	(0.21)		(0.28)	(<0.01)	(<0.01)	(<0.01)		
MLP	-1.72	-1.09		4.25	3.56	0.65	2	1
	(0.09)	(0.28)		(<0.01)	(<0.01)	(0.52)		
NAR	-4.50	-4.50	-4.25		-3.42	-4.33	0	5
	(<0.01)	(<0.01)	(<0.01)		(<0.01)	(<0.01)		
GA	-4.98	-5.04	-3.56	3.42		-4.73	1	4
	(<0.01)	(<0.01)	(<0.01)	(<0.01)		(<0.01)		
MEAN	-4.29	-3.59	-0.65	4.33	4.73		2	2
	(<0.01)	(<0.01)	(0.52)	(<0.01)	(<0.01)			

h= 4	LoLiMoT	AR	MLP	NAR	GA	MEAN	+	−
LoLiMoT		1.92	2.73	3.50	4.29	3.53	5	0
		(0.05)	(0.01)	(<0.01)	(<0.01)	(<0.01)		
AR	-1.92		1.58	3.44	4.27	2.64	3	1
	(0.05)		(0.11)	(<0.01)	(<0.01)	(0.01)		
MLP	-2.73	-1.58		3.18	2.41	-0.59	2	1
	(0.01)	(0.11)		(<0.01)	(0.02)	(0.55)		
NAR	-3.50	-3.44	-3.18		-2.57	-3.41	0	5
	(<0.01)	(<0.01)	(<0.01)		(0.01)	(<0.01)		
GA	-4.29	-4.27	-2.41	2.57		-3.88	1	4
	(<0.01)	(<0.01)	(0.02)	(0.01)		(<0.01)		
MEAN	-3.53	-2.64	0.59	3.41	3.88		2	2
	(<0.01)	(0.01)	(0.55)	(<0.01)	(<0.01)			

h=12	LoLiMoT	AR	MLP	NAR	GA	MEAN	+	−
LoLiMoT		1.89	0.97	5.71	4.34	3.18	4	0
		(0.06)	(0.33)	(<0.01)	(<0.01)	(<0.01)		
AR	-1.89		0.26	5.73	4.49	2.22	3	1
	(0.06)		(0.80)	(<0.01)	(<0.01)	(0.03)		
MLP	-0.97	-0.26		4.50	2.99	0.16	2	0
	(0.33)	(0.80)		(<0.01)	(<0.01)	(0.87)		
NAR	-5.71	-5.73	-4.50		-2.36	-5.77	0	5
	(<0.01)	(<0.01)	(<0.01)		(0.02)	(<0.01)		
GA	-4.34	-4.49	-2.99	2.36		-4.34	1	4
	(<0.01)	(<0.01)	(<0.01)	(0.02)		(<0.01)		
MEAN	-3.18	-2.22	-0.16	5.77	4.34		2	2
	(<0.01)	(0.03)	(0.87)	(<0.01)	(<0.01)			

h=24	LoLiMoT	AR	MLP	NAR	GA	MEAN	+	−
LoLiMoT		3.19	1.67	1.97	4.64	0.36	4	0
		(<0.01)	(0.10)	(0.05)	(<0.01)	(0.72)		
AR	-3.19		0.89	1.86	4.37	-0.95	2	1
	(<0.01)		(0.37)	(0.06)	(<0.01)	(0.34)		
MLP	-1.67	-0.89		1.68	1.50	-1.27	1	1
	(0.10)	(0.37)		(0.09)	(0.13)	(0.20)		
NAR	-1.97	-1.86	-1.68		-1.41	-1.78	0	4
	(0.05)	(0.06)	(0.09)		(0.16)	(0.07)		
GA	-4.64	-4.37	-1.50	1.41		-3.60	0	3
	(<0.01)	(<0.01)	(0.13)	(0.16)		(<0.01)		
MEAN	-0.36	0.95	1.27	1.78	3.60		2	0
	(0.72)	(0.34)	(0.20)	(0.07)	(<0.01)			

Note: see notes to Table 2.