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Identifying Periods of US Housing Market Explosivity

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Abstract

In this paper we set out to date-stamp periods of US housing price explosivity for the period 1830 – 2013. We make use of several robust techniques that allow us to identify such periods by determining when prices start to exhibit explosivity with respect to its past behaviour and when it recedes to long term stable prices. The first technique used is the Generalized sup ADF (GSADF) test procedure developed by Phillips, Shi, and Yu (2013), which allows the recursive identification of multiple periods of price explosivity. The second approach makes use of Robinson (1994)'s test statistic, comparing the null of a unit root process against the alternative of specified orders of fractional integration. Our analysis date-stamps several periods of US house price explosivity, allowing us to contextualize its historic relevance.

Keywords:
GSADF, Bubble, Structural Breaks, Random Walk, Explosivity
JEL: C22, G15, G14

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1. Introduction

The steep rise and subsequent fall of US house prices in the late 2000s have been the subject of much debate over the last few years. This follows largely from its role as underlying asset class to many of the derivative instruments

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that contributed to the unprecedented downward spiral of the fragile global financial system in 2008. Most economists agree that the housing market in the US, and in several other countries globally, for a myriad of reasons, experienced historically unprecedented asset price bubbles leading up to the global financial crisis. This made the market for mortgages especially attractive to home buyers and lenders alike, reinforcing the rapidly rising house prices and contributing to a booming industry destined for a reversal.

Property markets and residential houses, in particular, constitute a key asset class to the portfolio of most households worldwide. Abrupt movements of house prices, therefore, have a very real impact on households' abilities to consume and save. This in turn significantly impacts the economy's production and job creation capacity. As such, policies that curb unstable and bubble-like expansions in prices of houses in the economy could be considered a core policy objective, as sharp and sudden corrections in such prices could dramatically impact general price stability in the economy.

Since Shiller (1980) introduced the idea that prices of assets could deviate significantly from their underlying fundamentals (however defined), a large literature has emerged that aims to explain, document and even suggest preventions for asset price bubble formations. Although some efficient market proponents dismiss such notions, most accept that high transaction costs and limits to short selling could indeed lead to prices diverging from fundamental levels. As noted in Glaeser, Gyourko, and Saiz (2008), e.g., such market failures that hamper the ability of markets to correct price inefficiencies is particularly applicable to housing markets, where transaction costs are very high and short selling exceptionally difficult. This implies periods of price inefficiencies, and in particular periods of bubble-like behaviour, could feasibly exist with relatively little scope for arbitrage.

Our aim in this paper is to identify periods of bubble-like house price expansions over the last two centuries for the US market. This will serve to put the most recent bubble episode into historical perspective, and shed light on past price trends. The key research question is how to tell when rapidly rising house prices constitute a bubble. Case and Shiller (2003) defines a housing bubble as being driven by home buyers who are willing to pay inflated prices for houses due to their expectations that houses will keep experiencing unrealistic appreciation in the future. This notion might be based on high expected returns, with the "dividend" portion of holding the asset being the value of residing in the residence (or the rental income earned), with the capital gain the expected rising price of the home. In fact, both can be expected to experience periods of rapidly rising prices in the short run, which can fuel the demand for home-buyers and mortgage originators alike, as the value of the underlying asset rises. But, as seen in the US market in 2007, external factors might lead to costly corrections with very real economic impacts felt across income divides. Indeed, house prices may also experience such costly corrections as a result of deteriorating macroeconomic factors even though it might not have experienced a rapid increase before, or equivalently experience a gradual downward correction with little or no noticeable real costs. Our objective is not to estimate the costs or consequences of these periods of explosive price build-ups, but merely to document and contextualize their historic occurrences.

Our first challenge is identifying a fundamental level for house prices. As we face a lack of historical data on measures that have prviously been used to define such a level for house prices (including rental prices, construction costs and gross margins to home builders, as suggested by, among others, Himmelberg, Mayer, and Sinai (2005) and Glaeser et al. (2008))[1], we use another broad measure to define a level to which prices converge. Our premise is that house price movements tend, in the long run, to display stationary behaviour relative to broad price movements in the economy. We thus label periods of positive deviations from such stationarity for sustained periods as episodes of price explosivity. This can be motivated conceptually that during periods where house prices rise at a significantly higher rate than general prices in the economy, we can feasibly expect it to be experiencing inflationary pressures resembling explosive behaviour.

We then make use of two robust and efficient techniques that allow us to date-stamp periods of explosivity of these measures. The first technique that we will use is the Generalized sup ADF (GSADF) test procedure developed by Phillips, Shi, and Yu (2013), which is a recursive right-tailed unit root testing procedure that allows the identification of multiple periods of price explosivity. The second approach makes use of Robinson (1994)'s test of unit roots against the alternative of specified orders of fractional integration. We use the approach developed by Balcilar, Ozdemir, and Cakan (2015), which extends Robinson (1994)'s test statistic, to allow the identification of multiple periods of deviations from unit root behaviour in the presence

[1]We also do not directly account for different interest rate regimes, as our focus remains on the historical time-series behaviour of house prices.

of multiple endogenously determined structural breaks at unknown dates. This approach also provides the added benefit of testing a broader range of persistence than that which is measured using the unit root alternative in the first test. Using these techniques, we identify several periods of explosivity for real US house prices. We also find that unit roots exists for the full sample even when controlling for the existence of structural breaks, validating the first approach.

Our paper is structured as follows: section 2 discusses literature relevant to our study. Thereafter, section 3 outlines the methodologies used to identify periods of explosivity of US house prices, while section 4 describes the data used in the study. Section 5 discusses our findings, while section 6 concludes our study.

2. Review of relevant literature

Accurately documenting the inflationary build up of asset prices has long interested economists and policy makers alike. A vast literature has emerged that have tried to identify and explain the occurence of asset price bubbles, leading to often divergent views on suitable policy responses following its potential detection (c.f. Gürkaynak (2008) for an in-depth discussion of the performance of various bubble detection techniques). Often the difficulty in testing for the presence of bubble-like behavior in asset price series lie in correctly identifying and date-stamping multiple periods of explosivity. Traditional unit root and co-interation tests aimed at identifying such periods (as e.g. proposed by Diba and Grossman (1988)), fail to identify the existence of bubbles that periodically collapse. Evans (1991), e.g., points out that ordinary stationarity tests remain exposed to the possibility of identifying psuedo stationary behaviour when a series in fact displays periodically collapsing bubbles.

Various techniques have been proposed that allow the detection of multiple periods of collapsing speculative bubble in asset prices. Al-Anaswah and Wilfling (2011) and Lammerding, Stephan, Trede, and Wilfling (2013), e.g., use Markov-switching models to differentiate between regimes of price stability and price explosivity (the latter authors also use a robust Bayesian estimation procedure). Another class of techniques use a sequential unit root testing procedure developed by Phillips and Yu (2011) and Phillips, Wu, and Yu (2011), which built on the indirect stationarity tests suggested by Diba and Grossman (1984) and Hamilton and Whiteman (1985). As noted by

Bettendorf and Chen (2013), the key advantage of sequential identification procedures, particularly relevant to our analysis, is that it detects periods of explosivity despite potential misspecifications of the market fundamental process. In this study, we will make use of the generalized version of the sequential ADF tests, developed by Phillips, Shi, and Yu (2013) (PSY hereafter), which is robust to the identification of multiple collapsing bubble episodes. It has since gained ground in its broad empirical applications (c.f. inter alia Bettendorf and Chen (2013); Etienne, Irwin, and Garcia (2014); Caspi, Katzke, and Gupta (2015)) and allows consistent date-stamping for the origination and termination of multiple asset price bubbles.

A key challenge when using PSY's approach to identify asset price bubbles, is specifying the true definition of a fundamental level from which prices deviate. Typically, the return to holding the asset, in the form of dividend yields for equities (c.f. PSY, (2013)) and the convenience yield for commodities (c.f. Pindyck (1993); Lammerding et al. (2013); Gilbert (2010); Shi and Arora (2012)), is first defined in a pricing equation. Then a bubble component is specified, which, at times, displays explosive behaviour. Although several papers critique this identification of bubble components (e.g. Cochrane (2009); Pástor and Veronesi (2006); Cooper (2010) offer critical discussions on this), explosive or mildly explosive behaviour in asset price series indicate market exuberance during the inflationary phase of a bubble, a feature that can be uncovered from recursive testing procedures on time-series data (Phillips et al., 2013; Phillips and Magdalinos, 2007).

Caspi, Katzke, and Gupta (2015) also use the GSADF approach to identify periods where oil prices deviate from the general price level in the US, as well as levels of oil inventory supplies, respectively. Their use of these measures as proxies for the fundamental price of oil follow from a similar lack of data on historical oil price derivatives used to calculate the convenience yield. Instead, they study periods where the nominal price of oil displays periods of significant build-up relative to the general price level and stock of US oil supply, which both act as credible alternatives to the standard convenience yield.

The second approach that we will use in this study to identify periods of explosivity tests the null of a unit root process against the alternative of fractionally integrated orders which exceed one. Several studies have in the past used a long memory process to test for explosivity in asset price series using a test statistic developed by Robinson (1994) (e.g. Cuñado, Gil-Alana, and Gracia (2007); Gil-Alana (2003, 2008); Balcilar, Ozdemir, and Cakan

(2015). A key consideration in defining explosive periods are controlling for structural breaks, which, as highlighted by Perron (1989), may lead to the non-rejection of the unit-root hypothesis. Gil-Alana (2003) assumed known structural break dates in their analysis, while Gil-Alana (2008) employed a residuals sum squared approach where a single structural break date was allowed at an unknown time. Our approach follows that of Balcilar et al. (2015) in allowing multiple structural breaks at unknown dates. We then use Robinson (1994)'s LM test statistic to determine the fractional order of integration of the US house price series after controlling for endogenously determined level and trend shifts. We then recursively identify periods where the lower bound of the fractional order exceeds unity, and subsequently return to levels below unity, to allow us to identify explosive periods equivalent to those determined using PSY, (2013)'s GSADF approach. Both approaches are robust to multiple periods of periodically collapsing bubbles, less sensitive to the specific definition of the underlying fundamental process and able to provide recursive date-stamping of explosive periods in the underlying data.

Although not unique in its application to house price data[2], our long dated scope as well as our application of the techniques used in this paper to the housing bubble literature, is novel.

3. Methodological Discussion

The first technique that we use to label episodes of price explosivity builds on the work pioneered by Phillips and Yu (2011) and Phillips, Wu, and Yu (2011), and in particular the generalized form of the sup ADF (GSADF) proposed by Phillips, Shi, and Yu (2013). This method uses a flexible moving sample test procedure to consistently and efficiently detect and date-stamp periods where a price series displays a root exceeding unity. Bubbles are so identified in a consistent manner with false identifications seldom given even in modest sample sizes.[3] The test procedure suggested by PSY recursively implements an ADF-type regression test using a rolling window procedure. Suppose the rolling interval begins with a fraction r_1 and ends with a fraction

[2]Bourassa et al. (2001) provides a list of early studies on house price bubbles, with other notable studies including Case and Shiller (2003); Himmelberg et al. (2005); Glaeser et al. (2008), among many others.

[3]See PSY, (2013) for a deeper discussion and Monte-Carlo estimations testing the efficacy of this identification procedure.

r_2, with the size of the window given as $r_w = r_2 - r_1$. Then, let:

$$y_t = \mu + \delta . y_{t-1} + \sum_{i=1}^{p} \phi_{r_w}^{i} \delta . y_{t-i} + \epsilon_t \tag{1}$$

where μ, δ and ϕ are parameters estimated using OLS. We then test null of $H_0 : \delta = 1$ against the right sided alternative $H_1 : \delta > 1$. The number of observations used in equation 1 is then $T_w = [r_w T]$, where [.] is the integer part. The ADF statistic corresponding to equation 1 is thus denoted by $ADF_{r_1}^{r_2}$.

Building on this approach, PSY formulated a backward sup ADF test where the end point of the subsample remains fixed at a fraction r_2 of the entire sample, with the window size expanding from an initial fraction r_0 to r_2. This backward sup ADF (SADF) procedure can thus be defined as:

$$SADF_{r_2}(r_0) = sup_{r_1 \in [0, r_2 - r_0]} ADF_{r_1}^{r_2} \tag{2}$$

PSY then suggested repeatedly implementing the SADF procedure of equation 2 for each $r_2 \in [r_0, 1]$, leading to a generalized form (GSADF) written as:

$$GSADF(r_0) = sup_{r_2 \in [r_0, 1]} SADF_{r_2}(r_0) \tag{3}$$

The supremum form of the recursively estimated ADF is motivated by the observation that asset price bubbles generally collapse periodically.[4] In this scenario, the sup ADF test delivers efficient bubble detection capabilities where one or two bubbles emerge, with the generalized form performing well even in the presence of multiple bubble episodes.

The initial minimum fraction in the SADF approach of equation 2, $r_w = r_0$, is selected arbitrarily, keeping in mind the issue of estimation efficiency. Thereafter, we expand the sample window forward until $r_w = r_1 = 1$, the full sample, and we have a recursive estimate of ADF defined as $ADF_{r_k}, \forall k \in (r_0, r_1)$. From the sequence of ADF statistics (SADF) so produced, we can then identify the supremum value that can be used to test the null hypotheses of unit root against its right-tailed (mildly explosive) alternative by comparing it to its corresponding critical values. If the right tailed alternative to

[4] Evans (1991) pointed out that in samples with frequent bubble formations, conventional unit root tests have limited bubble detection power

the unit root null is thus accepted, we can infer mild explosivity of the series, indicated by δ_{r_1,r_2}.

The generalized form of this approach defined in equation 3, uses a variable window width approach which allows both the starting and ending points to change within a predefined range, $[r_0, 1]$. This allows the identification of multiple periods of explosivity and allows us to consistently date-stamp the starting and ending points. The starting points are identified as the periods, T_{r_e}, at which the backward sup ADF sequence crosses the corresponding critical value from below. The corresponding ending point to an explosive period is similarly defined as the period, T_{r_f}, where the backward sup ADF sequence crosses the critical value point from above.

We can formally define identified periods of explosivity using the GSADF approach as:

$$\begin{aligned}\hat{r_e} &= \inf_{r_2 \in [r_0, 1]} \left\{ r_2 : BSADF_{r_2} > cv_{r_2}^{\beta_T} \right\} \\ \hat{r_f} &= \inf_{r_2 \in [\hat{r_e}, 1]} \left\{ r_2 : BSADF_{r_2} > cv_{r_2}^{\beta_T} \right\}\end{aligned} \tag{4}$$

Where $cv_{r_2}^{\beta_T}$ is the $100(1 - \beta_t)\%$ critical value of the sup ADF statistic based on $[T_{r_2}]$ observations. We also set β_t to a constant value, 5%, as opposed to letting $\beta_T \to 0$ as $T \to 0$. The BSADF(r_0) for $r_2 \in [r_0, 1]$ is the backward sup ADF statistic that relates to the GSADF statistic by noting that:[5]

$$GSADF(r_0) = \underset{r_2 \in [r_0, 1]}{sup} \left\{ BSADF_{r_2}(r_0) \right\} \tag{5}$$

The second approach that we use also tests the right tailed alternative to a unit root null hypothesis, but unlike standard right-tailed tests, focuses on the fractional order of integration. The approach that we follow is similar to Balcilar et al. (2015), who built on the procedure developed by Robinson (1994) in determining the fractional order of integration. They also allow for the identification of multiple endogenously determined structural breaks in the form of level and trend shifts at endogenously determined dates. The identification approach is based on the procedure suggested by Gil-Alana (2008) and built on the principles suggested in Bai and Perron (1998). Balcilar et al. (2015) also construct statistical tests for the different orders of

[5]The recursive GSADF estimations were done using Caspi (2013)'s routine in Eviews.

fractional integration for each regime, using Robinson (1994)'s LM test to determine the most likely order of integration. To explain this procedure, consider the following multiple regression form:

$$y_t = \beta' z_t + x_t, \quad \forall t = 1, 2, ..., T \tag{6}$$

where y_t is the house price index series, β a $k \times 1$ vector of unknown parameters and z_t a $k \times 1$ vector of observable variables, which includes a constant, polynomials in time trends (t) and structural break dummies, depending on the deterministic structure imposed. As noted in Balcilar et al. (2015), the presence of such deterministic regressors does not affect the limiting null and local distribution of the Robinson test statistic.

We consider the general case where z_t includes a constant, a linear time trend and $m = 2k$ level, as well as trend shift dummies, $DLT^{tl}_{t,i} = (DL^{tl}_{t,i}, DT^{tl}_{t,i})'$ at the dates $i = T^{tl}_{b,1}, ..., T^{tl}_{b,k}$. We then set $DL^{tl}_{t,i} = 1$ if $t > T^{tl}_{b,i}$ and zero otherwise, and also $DT^{tl}_{t,i} = t - T^{tl}_{b,i}$ and zero otherwise. Here we will also follow the notation of Balcilar et al. (2015) by defining T_k as the set of disjoint break dates, $T_k = \{T^{tl}_{b,1}, ..., T^{tl}_{b,k}\}$. We also define $\beta' z_t$ as follows:

$$\beta' z_t = \mu + \delta.t + \sum_{i=1}^{k} \left(\Phi_i DL^{tl}_{t,i} + \Theta_i DT^{tl}_{t,i} \right) \tag{7}$$

with the regressor errors, x_t, assumed driven by the following process as:

$$(1 - L)^d x_t = u_t \tag{8}$$

with L the lag operator, u_t covariance stationary, integrated of order zero, $I(0)$, and having a spectral density function that is positive and finite at zero frequency. Allowing for a fractional order of integration in equation 8, implies that d can assume any value on the real line.

The model structure above is based on the least squares principle first proposed by Bai and Perron (1998). The estimation is carried out as follows: first, a grid of values for the fractional integration parameter, d, is chosen as, e.g., $d_0 = [0.00, 0.01, ..., 1.20]$. The least squares estimates of μ, δ, ϕ_i and θ_i in equation 8 are then obtained for each k-partition of $\{T_1, ..., T_k\}$, denoted as $\{T_k\}$, by minimizing the sum of squared residuals in the d_0 difference models. This implies, minimizing the following residuals sum of squares (RSS):

$$\sum_{t=1}^{T} (1 - L)^{d_0} \left(y_t - \mu - \delta.t - \sum_{i=1}^{k} \left[\phi_i.DL^{tl}_{t,i} + \Theta_i DT^{tl}_{t,i} \right]^2 \right) \tag{9}$$

over all value of $T_1, ..., T_k$, yielding the parameter estimates $\hat{\mu}, \hat{\delta}, \hat{\phi}_i$ and $\hat{\theta}_i, \forall i \in [1, ...k]$, and also the break dates, $\{\hat{T}_k\}$. We also employ Schwarz' (1978) Bayesian information criterion (BIC) to select the number of breaks, k, prior to running the procedure.[6] We then calculate the test statistic of Robinson (1994) for each value of d_0 in the grid, a procedure that can be summarized as follows (following again the notation of Balcilar et al. (2015)).

In order to test the null hypothesis:

$$H_0 : d = d_0 \tag{10}$$

Robinson (1994) developed the following score statistic:

$$\hat{r} = \left[\frac{\sqrt{T}}{\hat{\sigma^2}}\right] \sqrt{\hat{A}\hat{a}} \tag{11}$$

where

$$\hat{a} = -\frac{2\pi}{T}\sum_{j=l}^{T-1} \Psi(\lambda_j) g(\lambda_j; \eta); \quad \hat{\sigma^2} = \frac{2\pi}{T}\sum_{j=1}^{T-1} g(\lambda_j; \hat{\eta} I(\lambda_j))$$

$$\lambda_j = \frac{2\pi j}{T}; \quad I(\lambda_j) = \frac{1}{2\pi T}\left|\sum_{t=1}^{T} \hat{u}_t e^{i\lambda_j t}\right|$$

$$\hat{A} = \frac{2}{T}\left[\sum_{j=1}^{T-1} \Psi(\lambda_j)\Psi(\lambda_j)' - \sum_{j=1}^{T-1} \Psi(\lambda_j)\hat{\xi}(\lambda_j)' \times \left(\sum_{j=1}^{T-1} \hat{\xi}(\lambda_j)\hat{\xi}(\lambda_j)'\right)^{-1} \times \sum_{j=1}^{T-1} \hat{xi}(\lambda_j)\Psi(\lambda_j)'\right]$$

$$\hat{\xi}(\lambda_j) = \frac{\delta}{\delta\eta}\log(g(\lambda_j; \hat{\eta}); \quad \Psi(\lambda_j) = \mathrm{Re}\left\{\frac{\delta}{\delta\gamma}\log\phi(e^{-i\lambda_j}; \gamma_0)\right\} \tag{12}$$

with $I(\lambda_j)$ the periodogram of $\hat{u}_t$. Parameter estimates for $\hat{\eta}$ are derived from the Whittle Maximium Likelihood (WML) method:

$$\hat{\eta} = \mathrm{argmin}_{\eta\in\Lambda}\frac{2\pi}{T}\sum_{j=1}^{T-1} g(\lambda_j; \eta) I(\lambda_j) \tag{13}$$

[6] The number of breaks is selected by minimizing the criterion: $\mathrm{BIC}(k) = \ln[\frac{\mathrm{RSS}(\hat{T}_k)}{T-n}] + \frac{2n\ln(T)}{T}$

with $g(\lambda_j;\eta)$ the known function of the parametric spectral density of u_t. The model in equation 6 is completed by specifying a parametric form for u_t. In our analysis, we choose a general specification for u_t nested within an Autoregressive Moving Average (ARMA) model. This implies that by definition that x_t is characterized by a fractionally integrated ARMA (ARFIMA) model, which is a commonly used parametric specification for measuring long memory. The ARMA(p,q) model is denoted as:

$$\phi(L)u_t = \Psi(L)\varepsilon_t$$

while the ARFIMA(p,d,q) model for x_t can be written as:

$$\phi(L)(1-L)^d x_t = \Psi(L)\varepsilon_t \tag{14}$$

where ε_t is a white noise process with variance, σ^2, and $\phi(L) = 1-\sum_{j=1}^{p}\phi_j L^j$ and $\Psi(L) = 1-\sum_{j=1}^{q}\Psi_j L^j$ are polynomials in the lag operator L, with degrees of freedom p and q respectively. Furthermore, we assume that $\phi(Z)$ and $\Psi(Z)$ share no common roots and $\phi(Z)\neq 0$ and $\Psi(Z)\neq 0, \forall Z\leq 1$. The spectral density functions of these models, respectively, are given by:

$$f(\lambda;\sigma^2,\eta) = \frac{\sigma^2}{2\pi}\left|\frac{\Psi(e^{-i\lambda}}{\phi(e^{-i\lambda}}\right|, \quad \pi<\lambda\leq\pi \tag{15}$$

and

$$f(\lambda;\sigma^2,\eta) = \frac{\sigma^2}{2\pi}\left|\frac{\Psi(e^{-i\lambda}}{\phi(e^{-i\lambda}}\right|^2 \left|1-e^{-i\lambda}\right|^{-2d}, \quad \pi<\lambda\leq\pi \tag{16}$$

with η a $l\times 1$ vector of unknown parameters estimated by maximum likelihood, assuming that the orders p,q are known a priori.[7] Note also that the fractional parameter, d, is fixed under the null, thus equation 15 above is relevant to our empirical estimations. Our approach can thus be summarized as follows. We select a value d_0 in the grid $d_0^1 + i\Delta_d$, with Δ_d the grid increment and $i=1,...,s$. Then an initial disjoint break date, T_1, is selected and the residuals, $\hat{u_t} = (1-L)^{d_0}$, $\hat{x_t} = (1-L)^{d_0}y_t - \hat{\beta}'[(1-L)^{d_0}z_t]$, are thus obtained. This is then used to calculate the $\hat{r}$ statistic of equation 11, with

[7]For the ARMA model, $\eta = (\phi_1,...,\phi_p,\Psi_1,...,\Psi_q)'$ and for the ARFIMA model, $\eta = (d,\phi_1,...,\phi_p,\Psi_1,...,\Psi_q)'$, with $l=p+q+1$, implying that $g(\lambda_j;\eta) = |\frac{\Psi(e^{-i\lambda_j})}{\phi(e^{-i\lambda_j})}|$

break dates then updated using the Bai and Perron (1998) algorithm. These steps are then repeated until $\sum_{t=1}^{T} \hat{u}_t{}^2$ is minimized, and done for all the grid increments. At each step in the process, we minimize the $\text{RSS}(\hat{T}_k)$ for a given d_0, with the parameters $\hat{\beta}$ and nuisance parameters $\hat{\eta}$ estimated sequentially.

An approximate one-sided test of $H_0 : d = d_0$ is then rejected in favor of $H_a : d > d_0 (d < d_0)$ at the $100\alpha\%$ level when $\hat{r} > z_\alpha (\hat{r} < -z_\alpha)$, with α the probability that the standard normal distribution exceeds z_α. In the empirical implementation, we allow structural breaks in the full sample estimation. We use this procedure in the same fashion as the rolling window ADF regression of Phillips, Shi, and Yu (2013). In the rolling implementation, the sample interval begins with a fraction r_1 and ends with a fraction r_2, with the size of the window given as $r_w = r_2 - r_1$. We do not allow structural breaks in the rolling estimation since a small window size of r_w is unlikely to include structural break impacts.

4. Data Description

Our metric of interest in this study is the real house price (RHP) over the annual period of 1830-2013, with the start and end date being purely driven by data availability on house prices at the time of writing. The nominal house price is the Winans International U.S. Real Estate Index, which tracks the price of new homes back to 1830, obtained from the Global Financial Database. The nominal house price index is deflated by the Consumer Price Index (CPI) (and then multiplied by 100) to derive the real house price index. The CPI data is downloaded from the website of Robert Sahr (`http://oregonstate.edu/cla/polisci/sahr/sahr`). The RHP is then transformed into its natural logarithmic form.

The first step in using the GSADF date-stamping procedure is to apply the summary right-tailed GSADF tests to the series. Table 5 shows that for both series, at the 5% level (with the smallest window size of 15), we find that our GSADF test statistics exceed the 10% and 5% right-tailed critical values respectively, rejecting the hypothesis in favour of a root exceeding unity at some point. This provides evidence that RHP experienced periods of explosivity for the full sample. Using this approach to locate the bubbles, we compare the SADF statistic sequence with the 95% SADF critical value sequence, obtained using Monte Carlo simulations.[8] As can be seen from

[8]Details of this approach are contained in Phillips et al. (2013). The existence of

figure 2 in the appendix, RHP shows sustained growth in the post-war era, reaching its peak in 2005. Over the sample period, there were three episodes identified by the GSADF approach as explosive. Our fractional integration approach also provides evidence for the presence of several periods of explosivity for the RHP. These results and their economic relevance will be discussed in the next section.

5. Empirical Results

Figure 2 displays the results of the GSADF procedure over the sample period, with starting periods of explosivity labeled when the blue line (BSADF sequence) exceeds the red line (95% critical values), and ends where it dips below the red line. These periods of explosivity are summarized in table 1. We see that for the RHP series, there are three periods of explosivity with relatively short durations.[9] The first episode of explosivity was preceded by the five year depression following the panic of 1873, and saw the US Congress require a form of quantitative easing in the late 1870s.[10] This was followed by a spike in asset prices broadly, with real housing prices rising by 149% between 1878 and 1880. The second was between 1956 and 1957, where real house prices rose by over 43% between 1955 - 1957. This was driven by a decade of prosperity where the US economy grew significantly and employment were at all-time lows. The last episode identified is between 2004 and 2006. This follows a period where real house prices rose by roughly 26% from 2000–2006. The explosive episode identified was preceded by the Fed funds rate being lowered significantly[11], and characterized by sharply increasing house prices, large scale deregulation of institutions able to provide mortgage products, and a proliferation of investment vehicles designed by leveraged institutions to magnify the property market returns. This culminated in a period of unprecedented expansion in mortgage creation and housing price increases.

possible structural breaks in the series would merely serve to strengthen the argument for roots exceeding unity, and so we do not control for such events here.

[9] As noted by Phillips, Wu, and Yu (2011), periods of explosivity of short lengths should be excluded, which in our study we cut-off at a minimum of 2 periods for explosivity.

[10] The Bland-Allison Act of 1878 saw the US Congress require Treasury to buy up silver and in so doing inject liquidity into the economy.

[11] The Fed funds rate was lowered from 6.5% to 1.75% in 2001, following fears of a deflationary trap following the DotCom crash.

Table 1: GSADF explosive periods: RHP

Sample : 1830 - 2013		
Included observations: 184		
Starting Date	Ending Date	Duration (Years)
1879	1880	2
1956	1957	2
2004	2006	3

The next technique used in this study to label periods of explosiveness is the procedure proposed by Balcilar et al. (2015). The estimation was carried out as follows: for each chosen value of d we use the statistic for $\hat{r}$, given in equation 11, to test whether the fractional parameter, d, exceeds 1. This would be indicative of an explosive period, making it comparable to the sequential unit root tests above. We first test for various fractional orders d in the full ample. In our estimation for, the full sample we use two deterministic structures for z_t, with $z_{1,t}$ corresponding to a constant and trend, and $z_{2,t}$ corresponding to the general case in equation 7. The estimation procedure detailed in section 3, identified two endogenously determined linear trend and level breaks (denoted $DT_{t,i}^{t,l}$ and $DL_{t,i}^{t,l}$, respectively), which occurred at 1877 and 1954. These breaks correspond to periods of explosivity defined using the GSADF approach. The procedure is then used in a rolling estimation fashion with fixed window size of $r_w = 15$. Rolling estimation does not allow structural brea dummies, since a small window size does not suffer from structural break impacts.

The fit of the structural break model for the full sample can be viewed in figure 3 in the appendix. As can be seen, the model tracks the broad trend of the data rather well. Table 2 below provides the estimated full sample fit of the structural break model, using deterministic structure $z_{2,t}$.

Table 2: Estimates of deterministic and structural parameters

Constant	8.9579*** (0.0525)	9.4866*** (0.073)
Trend	0.0149*** (0.0005)	-0.0096*** (0.003)
$DL_{t,1}^{t,l}$		9.944*** (0.114)
$DT_{t,1}^{t,l}$		0.003** (0.001)
$DL_{t,2}^{t,l}$		8.963*** (0.295)
$DT_{t,2}^{t,l}$		0.015*** (0.002)
MA(1)	-0.032 (0.074)	0.043 (0.074)
MA(2)	0.016 (0.074)	0.176** (0.073)
BIC	-1.014	-1.337
$\hat{\sigma}$	0.355	0.248

Notes:
The table reports the parameter estimates of the model defined in equation 6 and explained thereafter, at minimum absolute values of the $\hat{r}$ statistic given in equation 11. Standard errors of the estimates are given in parentheses. ***, ** denote significance at 1% and 5% levels, respectively. $\hat{\sigma}$ is the standard error of the estimate and BIC the Bayesian Information Criterion.

From the table above, we see that nearly all of the parameters for the second model structure, $z_{2,t}$, are significant. The significant structural break dummy estimates confirm the existence of significant breaks in both trend and levels of the RHP series at 1877 and 1954.

In order to validate the use of the GSADF procedures earlier (as structural breaks could lead to the shifting up of orders of integration), we also include $z_{1,t}$'s estimates in table 3 below. From it we see firstly that when not controlling for the structural breaks, the lower bound of significance for the fractional order of integration estimate exceeds unity at the 1% level. When controlling for the structural breaks using $z_{2,t}$, we see non-rejection covers the range 0.94 to 1.00 at the 5% level, and 0.92 to 1.01 at the 1% level.[12] This indicates that there is strong evidence that RHP experienced periods of explosivity, when comparing the null of a unit root to the more flexible test of a fractional order of integration, even when not controlling for structural breaks. This validates the use of PSY, (2013)'s approach, as it indicates that

[12] Despite not rejecting range of values above 1 at the 1% level, it is clear that the lower bound is at the very least highly persistent and close to unity.

such breaks do not significantly account for explosivity in the full sample for RHP.

Table 3: Fractional integration estimations using Robinson (1994)'s statistic

d_0	$z_{1,t}$	$z_{2,t}$	d_0	$z_{1,t}$	$z_{2,t}$
0.81	21.32*,†	8.88*,†	1.01	0.64	-1.95†
0.82	19.60*,†	8.12*,†	1.02	0.09	-2.30*,†
0.83	17.99*,†	7.38*,†	1.03	-0.44	-2.63*,†
0.84	16.48*,†	6.67*,†	1.04	-0.94	-2.95*,†
0.85	15.06*,†	5.99*,†	1.05	-1.42	-3.26*,†
0.86	13.73*,†	5.34*,†	1.06	-1.87†	-3.55*,†
0.87	12.47*,†	4.71*,†	1.07	-2.30*,†	-3.84*,†
0.88	11.29*,†	4.10*,†	1.08	-2.71*,†	-4.11*,†
0.89	10.17*,†	3.52*,†	1.09	-3.09*,†	-4.37*,†
0.9	9.12*,†	2.96*,†	1.1	-3.46*,†	-4.61*,†
0.91	8.12*,†	2.42*,†	1.11	-3.81*,†	-4.85*,†
0.92	7.18*,†	1.90†	1.12	-4.15*,†	-5.08*,†
0.93	6.29*,†	1.40†	1.13	-4.46*,†	-5.30*,†
0.94	5.45*,†	0.92	1.14	-4.76*,†	-5.51*,†
0.95	4.65*,†	0.46	1.15	-5.05*,†	-5.71*,†
0.96	3.89*,†	0.02	1.16	-5.32*,†	-5.90*,†
0.97	3.17*,†	-0.41	1.17	-5.58*,†	-6.09*,†
0.98	2.49*,†	-0.82	1.18	-5.83*,†	-6.26*,†
0.99	1.84†	-1.21	1.19	-6.06*,†	-6.43*,†
1	1.22	-1.59	1.2	-6.28*,†	-6.59*,†

Notes:
* and † indicate the non-rejection at the 1% and 5% levels, respectively, when comparing the $\hat{r}$ statistic to the standard normal critical values for a one sided test.
$z_{1,t}$ indicates a deterministic structure with no structural breaks, while $z_{2,t}$ has two endogenously identified linear trend and level breaks.

In order to date-stamp periods of explosivity using this approach, we employ a rolling window procedure to calculate the $\hat{r}$ statistic. We use a fixed length window size of 15 sequentially from the beginning to the end of the sample, adding a single observation and dropping the last at each step. We then calculate at each step a range of $\hat{r}$ statistics, enabling us to estimate a lower and upper bound limit for d (using a one sided test with 5% significance level). The benefit of using this approach to identify periods of explosivity is that, firstly, it allows for a changing structure of the underlying data, and secondly it is robust to possible structural breaks. This implies we use the rolling window identification technique on the $z_{1,t}$ deterministic form, as opposed to the form accounting for the breaks explicitly. Although there are differing views on the appropriate size of such fixed window techniques[13], our chosen window size reflects our desire to optimize the representativeness of the model, particularly as we identified two breaks in the series. Figure 1 below shows our rolling window estimations.

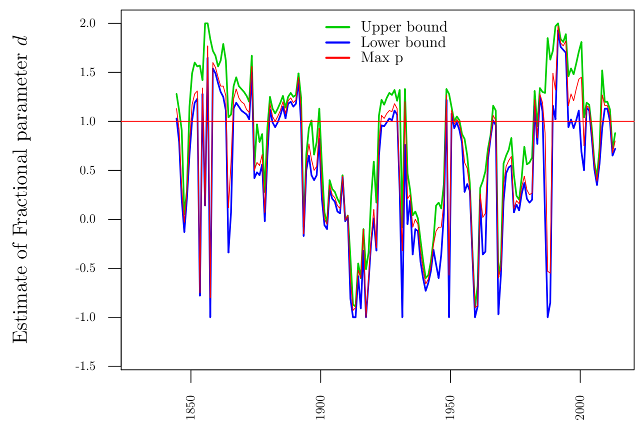

Figure 1: Rolling estimations of the $\hat{r}$ statistics

From figure 1, we identify periods of explosivity as starting when the lower bound (blue line) cross 1, and ends when it dips below 1. Table 4 summarizes the periods of explosivity so identified. As before, we ignore episodes shorter than 1 period in duration, while also excluding periods of potential negative explosivity, as our focus is on price build-ups.

[13]C.f. Pesaran and Timmermann (2005) for a deeper discussion.

Table 4: Rolling $\hat{r}$ explosive periods: RHP

Sample : 1830 - 2013		
Included observations: 184		
Starting Date	Ending Date	Duration (Years)
1850	1852	3
1858	1863	6
1866	1873	8
1926	1929	4
1984	1985	2
1998	1999	2
2002	2003	2
2009	2011	2

From table 4, we see that the fractional integration rolling window approach offers greater insight into periods of explosiveness during the 1800s, particularly as there is a much shorter burn-in period. We see, e.g., several periods of explosiveness in real house prices during the 1849 – 1855 California gold rush[14], which saw an increase in real house prices of over 70% during this period.[15] The US gold rush continued until 1864, during which time another period of explosivity can be identified towards the latter part. The next period of RHP explosivity is labeled between 1866 – 1873, right after the US civil war which ended in 1865. Prices during this phase peaked in 1867, 81% higher than during the war in 1864. The next episode of explosivity identified is from 1926 – 1929, during which time real house prices rose by over 48%. This coincided with unprecedented asset price inflation across nearly all US asset classes. Real house prices peaked in 1928, and were down 85% by 1932, following the start of the Great Depression in 1929.

This approach then identifies two short-lived periods of explosivity during the mid 1980s and late 1990s. The first episode transpired in the build up to what is today known as the Savings and Loan crisis, which started in 1986, and saw real prices rise by roughly 30% from 1984–1987. This was caused in part by large scale deregulation of lending standards and a reduction in capital reserve requirements in the US, which both served to drive large scale

[14] C.f. Santos, R. L. (1998). The Gold Rush of California: A Bibliography of Periodical Articles. *Cass Publications*.

[15] RHP peaked in 1853 at 194% above the level in 1848.

credit creation, particularly in financing mortgages. The 1990s saw RHP first decline substantially (after peaking in 1989, it fell by roughly 21% by 1993), while picking up in the late 1990s and reaching its 1989 peak again in 2001. Real prices then surged in the early 2000s, peaking in 2004 at 27% higher than in 2000. The RHP correction came after 2006, with a turnaround in RHP between 2009 – 2011 identified as a significant rise in RHP.

6. Conclusion

This paper set out to identify periods of US house price explosivity from 1830 – 2013. In order to identify house price fundamentals, we make use of the general price level (measured as the US CPI index). The implicit assumption thus made is that house prices tend to reflect general movements in prices across the economy. Large deviations from past levels could therefore be considered as explosive in the short term as it could feasibly lead to higher allocation towards houses as assets experiencing high capital growth. This, in turn, feeds into more demand and even higher prices, potentially driving an episode of unsustainable asset price increases, particularly as a result of factors inherent to property purchases (such as typically high transaction costs and low ability to short-sell) that make it uniquely prone to bubble-type episodes. Although other measures have been suggested for use as fundamentals, we are constrained by data availability for our long dated sample.[16]

The first technique used to identify periods of explosivity, is the recursive GSADF test suggested by Phillips et al. (2013). This test allows the effective date-stamping of periodically collapsing bubble-like periods, allowing us to label several historical periods of significant real house price build-ups. For the RHP measure, we define three short periods of explosivity, during the late 1800s, mid 1950s and the mid 2000s.

The second measure used to test right tailed alternatives to unit root testing, focuses on the fractional order of integration, d. The procedure uses Robinson (1994)'s $\hat{r}$ statistic to define confidence bands for likely values of d. We also allow for the identification of multiple periods of endogenously determined structural breaks in the form of level and trend shifts at unknown dates. We then use a rolling window approach to date-stamp periods of likely

[16]Despite this, we maintain the appropriateness of these measures as proxying an essentially immeasurable fundamental level.

explosivity in the series, identified as periods where the lower bound of the 95% confidence interval of d exceeds unity. The periods so identified suggest several periods of explosivity during the 1800s, particularly surrounding the US gold rush, as well as immediately following the Civil War. Significant and unsustainable build-ups in real house prices are then also observed in the 1920s shortly before the Great Depression, the 1980s during the period preceding the S&L crisis, as well as during the late 1990s and early and late 2000s. Our results suggest that the more flexible, long memory approach of using fractional integration to test the alternative hypothesis, provides a richer set of dates of where prices likely deviated from mean reversion toward aggregate prices in the US.

In summary, our analysis provides a thorough investigation of the time-series characteristic of US house prices over the last two centuries, novel in its coverage as well as use of fractional integration in determining house price explosivity.

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7. Appendix

Table 5: Right Tailed ADF Test

Sample : 1830 2013			
Included observations: 184			
Lag Length: Fixed, lag=0			
Window size: 15			
H_0: RHP has a unit root			
		t-Statistic	Prob.
GSADF		0.050	0.033
Test critical values:	99% level	0.622	
	95% level	-0.167	
	90% level	-0.519	

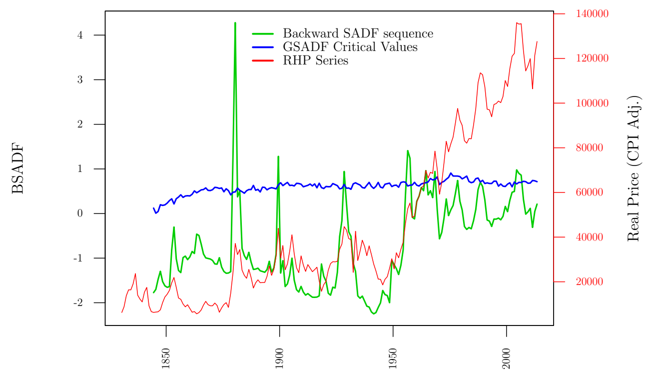

Figure 2: Backward SADF procedure: Real House Price

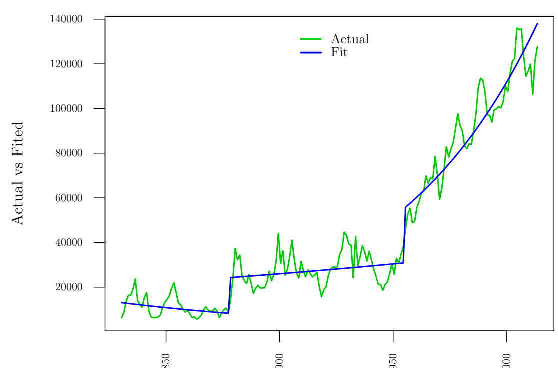

Figure 3: Actual versus Fitted values of fractional integration model