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Taxable Sales Using Coincident and Leading  
Employment Indexes**

*Mehmet Balcilar, Rangan Gupta,  
Anandamayee Majumdar, Stephen Miller*

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***Department of Economics***  
***Eastern Mediterranean University***  
***Famagusta, North Cyprus***

# **Forecasting Nevada Gross Gaming Revenue and Taxable Sales Using Coincident and Leading Employment Indexes**

Mehmet Balcilar  
Department of Economics  
Eastern Mediterranean University  
Famagusta, NORTHERN CYPRUS

Rangan Gupta  
Department of Economics  
University of Pretoria  
Pretoria, 0002, SOUTH AFRICA

Anandamayee Majumdar  
School of Mathematical & Statistical Sciences  
Arizona State University  
Tempe, AZ 85287-1804 USA

Stephen M. Miller\*  
Department of Economics,  
University of Nevada, Las Vegas  
Las Vegas, Nevada, 89154-6005 USA

## **Abstract**

This paper provides out-of-sample forecasts of Nevada gross gaming revenue and taxable sales using a battery of linear and non-linear forecasting models and univariate and multivariate techniques. The linear models include vector autoregressive and vector error-correction models with and without Bayesian priors. The non-linear models include non-parametric and semi-parametric models, smooth transition autoregressive models and artificial neural network autoregressive models. In addition to gross gaming revenue and taxable sales, we employ recently constructed coincident and leading employment indexes for Nevada's economy. We conclude that non-linear models generally outperform linear models in forecasting future movements in gross gaming revenue and taxable sales.

**Keywords:** Forecasting, Linear and non-linear models, Nevada gross gaming revenue, Nevada taxable sales

**JEL classification:** C32, R31

*\* Corresponding author*

## **1. Introduction**

The Great Recession in the US creates significant challenges for state governments as they plan for future budgets. That is, most state governments live under a balanced operating budget from year-to-year. Forecasting state revenues, therefore, becomes a significant task.

Nevada faces the most severe decline in state tax revenue in its history. In prior recessions, Nevada's economy barely noticed the national recession and continued to grow, or at least experienced a much lower decline in its economy than the nation as a whole. The Great Recession that witnessed significant declines in the leisure and hospitality, construction, and finance, insurance, and real estate sectors caused Nevada to experience its worst recession ever.

Gross gaming revenue and taxable sales comprise two major components of Nevada's tax base, generating sales and use taxes, and gaming taxes respectively. In fiscal 2008, sales and use taxes constituted 35.6 percent of total Nevada taxes while gaming taxes constituted 29.0 percent. On total general fund revenue, which adds licenses, fees, fines and other revenues to taxes, sales and use taxes comprise 32.3 percent and gaming taxes comprise 26.3 percent. In sum, well over 50 percent of Nevada revenues come from the taxable sales and gross gaming revenue tax bases.

Thus, this paper provides out-of-sample forecasts of Nevada gross gaming revenue and taxable sales using a battery of linear and non-linear forecasting models and univariate and multivariate techniques. Linear models include vector autoregressive (VAR), Bayesian VAR (BVAR), vector error-correction (VEC), and Bayesian VEC (BVEC) models. Non-linear models include semi-parametric (SP), non-parametric (NP), smooth transition autoregressive (STAR), and artificial neural network (ANN) models. In addition to the two components of Nevada's tax base, this paper also employs recently constructed CBER-DETR Nevada Coincident and Leading Employment indexes to capture the state of the Nevada economy.

We organize the rest of the paper as follows. Section 2 discusses the construction of the Nevada coincident and leading Employment indexes. Section 3 reviews the existing literature on forecasting Nevada taxable sales and gross gaming revenue. Section 4 outlines the various methodologies used to forecast Nevada gross gaming revenue and taxable sales – VAR, BVAR, VEC, and BVEC models, semi-parametric and non-parametric models, smooth transition autoregressive models, and artificial neural network models. Section 5 describes the data and reports the results of the various linear and non-linear forecasting methods. Section 6 concludes.

## **2. Coincident and Leading Employment Indexes for Nevada**

Coincident indexes include a number of economic series that collectively represent the current state of the economy. Each series in a coincident index contains some information about the turning points in the business cycle. Since series do not all show the same turning points, a coincident index provides a collective call on the business cycle. This averaging process produces better information about cyclical turning points than any one of the individual series in the index can generate on their own.

Leading indexes provide valuable information about the future path of the economy, combining information from several economic series and collectively forecasting future movements in the economy. As with coincident indexes, each series provides some information but it is unlikely that the individual series will show identical turning points. The combined information in leading indexes produces better predictions about future turning points.

Dua and Miller (1995) construct Connecticut coincident and leading employment indexes following well-developed procedures used by the Department of Commerce and described in U.S. Department of Commerce (1977, 1984) and in Niemira and Klein (1994). These procedures

adopt methods developed by National Bureau of Economic Research researchers Geoffrey H. Moore and Julius Shiskin in the 1950s.

Several characteristics of a time series are evaluated to select the components of a composite index. The most important of these is cyclical timing determined by the consistency with which the cyclical turning points in a series coincide with or lead the business cycle turns. Other factors include the periodicity of the data, their reliability, and the promptness with which they are available. The components are standardized to prevent the more volatile series from dominating the index.

Dua and Miller (1995) base their indexes on employment-related time series only due to data availability constraints. The components of the indexes are available monthly with a short time lag and are reliable. Since employment conditions generally mirror overall economic activity, the coincident and leading indexes serve as measures of current and future economic activity, respectively.

The Nevada coincident employment index comprises four individual employment-related series that track current employment activity - the total unemployment rate (inverted), the insured unemployment rate (inverted), nonfarm employment, and total (household) employment. The index, therefore, combines information from different sources. Total (household) employment and the unemployment rate are based on a survey of about 600 Nevada households. The insured unemployment rate, on the other hand, comes from the data on unemployment insurance claims filed with the state. Finally, nonfarm employment is based on a survey of employers by the state.

The Nevada leading employment index includes six components that predict employment activity - the initial claims for unemployment insurance (inverted), the short-duration (less than

15 weeks) unemployment rate (inverted), housing permits, commercial permits, construction employment, and the real Moody's Baa interest rate (inverted). Each variable has some intuitive appeal. The initial claims for unemployment insurance is one of the first steps taken by someone who loses his/her job. So, this variable quickly reflects job market changes. The short-duration unemployment rate measures changes in those unemployed for 15 weeks or less. This variable also quickly reflects changes in the job market. Housing and commercial permits captures the intention to build in the near future, which closely relates to construction employment. Finally, the Baa interest rate, although a national variable, provides useful leading information for the state (Banerji, Dua, and Miller 2006).

Figures 1 and 2 report the coincident and leading indexes using data through December 2009. At that time, it looked like the coincident and leading indexes bottomed in October 2009. All data are seasonally adjusted and come from DETR, CBER, and the Federal Reserve Bank of St. Louis FRED® data. The description of the construction method is posted at <http://cber.unlv.edu/nvindices.pdf>. Data availability restricts our coverage in the two indexes to monthly series beginning in January 1976. The data series for household employment, nonfarm employment, the unemployment rate, initial claims, and the real Moody's Baa bond rate all begin in January 1976. Housing permits and the insured unemployment rate begins in January 1980 and March 1987, respectively. Commercial permits, construction employment, and the short-duration unemployment rate begin in January 1988, January 1990, and January 2001, respectively. Thus, the coincident index uses three series through March 1987, when we add the insured unemployment rate. The leading index begins with two series and adds housing permits in January 1980, commercial permits in January 1988, construction employment in January 1990, and finally, the short-duration unemployment rate in January 2001.

In sum, the Nevada coincident employment index contains four individual series that gauge current economic activity. By construction, it includes more information than that in a single measure of economic activity such as the unemployment rate. Likewise, the leading employment index contains six individual series that predict future economic activity.

### **3. Literature Review**

Several papers model and forecast Nevada gross gaming revenue. Cargill and Eadington (1978) develop simple models of Nevada gross gaming revenue – structural and autoregressive integrated moving average (ARIMA). They actually investigate gross gaming revenue for three different regions in Nevada – Las Vegas, Reno-Sparks, and South Lake Tahoe. The single equation structural models include California personal income and dummy variables for the 1973-74 energy crisis and recessions. Using quarterly data, the ARIMA models employ one regular difference and one seasonal difference with one autoregressive term.<sup>1</sup> They estimate the ARIMA model from 1955:Q1 to 1974:Q4 and forecast 1975:Q1 to 1977:Q4, reporting the errors as a percent of the actual values.<sup>2</sup>

Cargill and Morus (1988) develop an eight variable Bayesian vector autoregressive (BVAR) model of the Nevada economy. The model includes four national drivers – real GNP, the GNP deflator, employment, and the 4-6 month commercial paper rate -- and one California driver – employment -- along with three Nevada variables – gross gaming revenue, taxable sales,

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<sup>1</sup> Eisendrath, Bernhard, Lucas, and Murphy (2008) use intervention analysis to determine the effect of the terrorist attack on 9/11 on Nevada gross gaming revenue. They conclude that the recovery from the attack occurred rather quickly with most recovery occurring in five months and complete recovery within two years.

<sup>2</sup> Traditionally, for forecasting purposes, time-series models generally perform as well as or better than dynamic structural econometric specifications. Zellner and Palm (1974) provide the theoretical rationalization. Any dynamic structural model implicitly generates a series of univariate time-series models for each endogenous variable. The dynamic structural model, however, imposes restrictions on the parameters in the reduced-form time-series specification. Dynamic structural models prove most effective in performing policy analysis, albeit subject to the Lucas critique. Time-series models prove most effective at forecasting. That is, in both cases errors creep in whenever the researcher makes a decision about the specification. Clearly, more researcher decisions relate to a dynamic structural model than a univariate time-series model, suggesting that fewer errors enter the time-series model and allowing the model to produce better forecasts.

and non-farm employment. From variance decompositions, they conclude that Nevada proves more isolated from national events than is California. Moreover, much of the movement in Nevada variables gets explained by Nevada's variables. For example, the variance of Nevada's employment helps to explain the variances of both Nevada's taxable sales and gross gaming revenue. And, the variance of Nevada's taxable sales helps to explain the variance of Nevada's employment. For forecasting purposes, they assume that the national variables are exogenous to the model. The forecasts from their model for Nevada's taxable sales and employment prove better than the naïve model while those for gross gaming revenue do not. They suspect that the adoption of the lottery in the middle of their forecast horizon drove the poor forecasts for gross gaming revenue.<sup>3</sup>

Shonkwiler (1992) develops a state-space model using the Kalman filter, which introduces stochastic parameter estimation, to forecast gross taxable gaming revenue in Nevada. The econometric procedure allows the slope of the trend in the data series to vary over time. Under various assumptions within this stochastic trend model, different ARIMA models emerge. The stochastic trend model out-performed the BVAR model on Cargill and Raffee (1990) in forecasting quarterly gaming revenue out of sample over 1988 to 1989, although both methods tended to overestimate gross gaming revenue.

#### **4. Methodology:**

This section describes the various methodologies used to forecast Nevada gross gaming revenue and taxable sales.

##### *4.1. VAR, VEC, BVAR, and BVEC Specifications*

Following Sims (1980), we write an unrestricted VAR model as follows:

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<sup>3</sup> Cargill and Raffee (1990) update the Nevada BVAR model to include Nevada personal income as well as to include a dummy variable for the California lottery.



$$(1) \quad y_t = A_0 + A(L)y_t + \varepsilon_t,^4$$

where  $y$  equals a  $(n \times 1)$  vector of variables, which in our case, includes four variables -- gross gaming revenue (GGR), taxable retail sales (TRS), the leading employment index (LI), and the coincident employment index (CI);  $A(L)$  equals an  $(n \times n)$  polynomial matrix in the backshift operator  $L$  with lag length  $p$ ,<sup>5</sup> and  $\varepsilon$  equals an  $(n \times 1)$  vector of error terms. In our case, we assume that  $\varepsilon \sim N(0, \sigma^2 I_n)$ , where  $I_n$  equals an  $(n \times n)$  identity matrix.

With cointegrated (non-stationary) series, we can transform the standard VAR model into a VEC model. The VEC model builds into the specification of the cointegration relations so that they restrict the long-run behavior of the endogenous variables to converge to their long-run, cointegrating relationships, while at the same time describing the short-run dynamic adjustment of the system. The cointegration terms, known as the error correction terms, gradually correct through a series of partial short-run adjustments.

More explicitly, for our four variable system, if each series  $y_t$  is integrated<sup>6</sup> of order one, (i.e.,  $I(1)$ ),<sup>7</sup> then the error-correction counterpart of the VAR model in equation (1) converts into a VEC model as follows.<sup>8</sup>

$$(2) \quad \Delta y_t = \pi y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \varepsilon_t,$$

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<sup>4</sup>  $A(L) = A_1 L + A_2 L^2 + \dots + A_p L^p$ ; and  $A_0$  equals an  $(n \times 1)$  vector of constant terms.

<sup>5</sup> Note that we estimate all the (V)AR models in the log levels of the variables. We use four lags as confirmed by the sequential modified LR test statistic, Akaike information criterion (AIC), and the final prediction error (FPE) criterion, obtained from an estimation of a stable VAR model. Stability, as usual, implies that no roots lie outside the unit circle.

<sup>6</sup> A series is integrated of order  $q$ , if it requires  $q$  differences to transform it into a zero-mean, purely non-deterministic stationary process.

<sup>7</sup> See LeSage (1999) and references cited therein for further details regarding the non-stationarity of most macroeconomic time series.

<sup>8</sup> See Dickey *et al.* (1991) and Johansen (1995) for further technical details.

where  $\pi = -[I - \sum_{i=1}^p A_i]$  and  $\Gamma_i = -\sum_{j=i+1}^p A_j$ .

VAR and VEC models typically use equal lag lengths for all variables in the model, which implies that the researcher must estimate many parameters, including many that prove statistically insignificant. This over-parameterization problem can create multicollinearity and a loss of degrees of freedom, leading to inefficient estimates, and possibly large out-of-sample forecasting errors. Some researchers exclude lags with statistically insignificant coefficients. Alternatively, researchers use near VAR models, which specify unequal lag lengths for the variables and equations.

Litterman (1981), Doan *et al.*, (1984), Todd (1984), Litterman (1986), and Spencer (1993), use a Bayesian VAR (BVAR) model to overcome the over-parameterization problem. Rather than eliminating lags, the Bayesian method imposes restrictions on the coefficients across different lag lengths, assuming that the coefficients of longer lags may approach more closely to zero than the coefficients on shorter lags. If, however, stronger effects come from longer lags, the data can override this initial restriction. Researchers impose the constraints by specifying normal prior distributions with zero means and small standard deviations for most coefficients, where the standard deviation decreases as the lag length increases. The first own-lag coefficient in each equation is the exception with a unitary mean. Finally, Litterman (1981) imposes a diffuse prior for the constant. We employ this “Minnesota prior” in our analysis, where we implement Bayesian variants of the classical VAR and VEC models.

Formally, the means and variances of the Minnesota prior take the following form:

$$(3) \quad \beta_i \sim N(1, \sigma_{\beta_i}^2) \text{ and } \beta_j \sim N(0, \sigma_{\beta_j}^2)$$

where  $\beta_i$  equals the coefficients associated with the lagged dependent variables in each equation of the VAR model (i.e., the first own-lag coefficient), while  $\beta_j$  equals any other coefficient. In sum, the prior specification reduces to a random-walk with drift model for each variable, if we set all variances to zero. The prior variances,  $\sigma_{\beta_i}^2$  and  $\sigma_{\beta_j}^2$ , specify uncertainty about the prior means  $\bar{\beta}_i = 1$ , and  $\bar{\beta}_j = 0$ , respectively.

Doan *et al.*, (1984) propose a formula to generate standard deviations that depend on a small numbers of hyper-parameters:  $w$ ,  $d$ , and a weighting matrix  $f(i, j)$  to reduce the over-parameterization in the VAR and VEC models. This approach specifies individual prior variances for a large number of coefficients, using only a few hyper-parameters. The specification of the standard deviation of the distribution of the prior imposed on variable  $j$  in equation  $i$  at lag  $m$ , for all  $i, j$  and  $m$ , equals  $S_I(i, j, m)$ , defined as follows:

$$(4) \quad S_I(i, j, m) = [w \times g(m) \times f(i, j)] \frac{\hat{\sigma}_i}{\hat{\sigma}_j},$$

where  $f(i, j) = 1$ , if  $i = j$  and  $k_{ij}$  otherwise, with  $(0 \leq k_{ij} \leq 1)$ , and  $g(m) = m^{-d}$ , with  $d > 0$ . The estimated standard error of the univariate autoregression for variable  $i$  equals  $\hat{\sigma}_i$ . The ratio  $\hat{\sigma}_i / \hat{\sigma}_j$  scales the variables to account for differences in the units of measurement and, hence, causes specification of the prior without consideration of the magnitudes of the variables. The term  $w$  indicates the overall tightness and equals the standard deviation on the first own lag, with the prior getting tighter as the value falls. The parameter  $g(m)$  measures the tightness on lag  $m$  with respect to lag 1, and equals a harmonic shape with decay factor  $d$ , which tightens the prior at longer lags. The parameter  $f(i, j)$  equals the tightness of variable  $j$  in equation  $i$  relative to

variable  $i$ , and by increasing the interaction (i.e., the value of  $k_{ij}$ ), we loosen the prior.<sup>9 10</sup>

#### 4.2. *Nonparametric and Semi-Parametric Models*

We now proceed with a nonparametric and semi-parametric regression approach for forecasting both gross gaming revenue and taxable retail sales. To ensure stationarity of the variables, we work with the growth rates, and not the actual levels, when fitting the models, and making the forecasts.<sup>11</sup> Eventually after the forecasts of the growth rates are made, the forecasts related to the actual levels are recovered once more.

First, we abbreviate all the variables that are used in these models. The variables used for modeling and initial forecasting, correspond to the growth rates of: GGR, TRS, LI, and CI, but for convenience of understanding, we use the same terminologies to address the growth rates as we do the levels. Thus, we abbreviate the growth rate of GGR as GGR too. We also use GGR1, GGR2, and GGR3 to denote the first, second, and third lags of the growth rate of GGR, respectively.<sup>12</sup> The rest of the cases follow accordingly.

We examine three competing models, and examine their forecasting abilities. These specifications occur as follows:

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<sup>9</sup> For an illustration, see Dua and Ray (1995).

<sup>10</sup> We estimate the (B)VAR and (B)VEC models, as well as the random-walk model, using the Econometrics Toolbox in MATLAB.

<sup>11</sup> Note that non-parametric and semi-parametric estimation, as well as smooth-transition-autoregressive and artificial-neural-network models, require that the variables are stationary to avoid spurious estimates. Hence, we convert all the variables to their monthly growth rates and test the converted series for stationarity by the Augmented-Dickey-Fuller (ADF), the Dickey-Fuller with GLS detrending (DF-GLS), the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS), and the Phillips-Perron (PP) tests. In other words, we find that all variables are  $I(1)$  in levels. The results are available upon request from the authors.

<sup>12</sup> Note that with the semi-parametric, non-parametric, and smooth transition autoregressive models, the artificial-neural-network models and the VECMs based on the growth rates of the variables, use three lags, since we estimate the VARs in levels of the variables with four lags.

**Model 1: Nonparametric full regression model (NPFR model)**

$$(5) \quad GGR = f(GGR_1, GGR_2, GGR_3, TRS_1, TRS_2, TRS_3, LI_1, LI_2, LI_3, CI_1, CI_2, CI_3) + \varepsilon_{GGR}$$

$$(6) \quad TRS = g(GGR_1, GGR_2, GGR_3, TRS_1, TRS_2, TRS_3, LI_1, LI_2, LI_3, CI_1, CI_2, CI_3) + \varepsilon_{TRS}$$

**Model 2: Nonparametric partial regression model (NPPR model)**

$$(7) \quad GGR = f(GGR_1, GGR_2, GGR_3) + \varepsilon_{GGR}$$

$$(8) \quad TRS = g(TRS_1, TRS_2, TRS_3) + \varepsilon_{TRS}$$

**Model 3: Semi-parametric full regression model (SPFR model)**

$$(9) \quad GGR = \alpha_0 + \alpha_1 GGR_1 + \alpha_2 GGR_2 + \alpha_3 GGR_3 \\ + f(TRS_1, TRS_2, TRS_3, LI_1, LI_2, LI_3, CI_1, CI_2, CI_3) + \varepsilon_{GGR}$$

$$(10) \quad TRS = \beta_0 + \beta_1 TRS_1 + \beta_2 TRS_2 + \beta_3 TRS_3 \\ + g(TRS_1, TRS_2, TRS_3, LI_1, LI_2, LI_3, CI_1, CI_2, CI_3) + \varepsilon_{TRS}$$

Here,  $f(\cdot)$  and  $g(\cdot)$  denote unknown functions that are estimated from the data. The  $\varepsilon_{GGR}$  and  $\varepsilon_{TRS}$  are mean-zero errors, with unchanged variance over the entire data set. The parameters  $\alpha_i$ ;  $\beta_i$ ;  $i = 1; 2; 3$  are constants estimated from the data. Therefore, the semi-parametric model can also be described as a partially linear nonparametric model.<sup>13</sup>

We checked the goodness of model fit using Bootstrap testing and found p-values close to 1 for the models used. When estimating the unknown functions  $f(\cdot)$  and  $g(\cdot)$  in case of the nonparametric models, a local linear regression using  $AIC_c$  bandwidth selection criterion was used. In this case, we also examined all the options for the choice of kernels, and found that the Gaussian kernel of order 2 worked the best yielding highest R-squared values and smallest MSE. The optimum bandwidth chosen by the software was used. In case of the semi-parametric modeling, we first computed data-driven bandwidths of the kernels to be used in the  $f(\cdot)$  and  $g(\cdot)$

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<sup>13</sup> We use the *np* package in *R* to carry out the regressions outlined above.

parts of the model since selection of bandwidth for lower levels of tolerance takes a very large proportion of time. We overrode the default tolerances, and set the tolerance levels at 0.1 for the search method as the objective function is well-behaved. The regression type was local constant, and not local linear, as local linear seems to yield smaller R-squared values. Again, for the  $f(\cdot)$  and  $g(\cdot)$  parts of the model, we used Gaussian kernels of order 2, because they yielded highest R-squared values and lowest MSE.

#### 4.3. *Smooth Transition Autoregressive Model Identification*

Recent empirical studies show that smooth-transition-autoregressive (STAR) models can successfully model economic time series that move smoothly between two or more regimes, e.g., recession to expansion. When considering the joint dynamic properties of gross gaming revenue, taxable retail sales, the leading index, and the coincident index, it is natural to consider multivariate STAR (MSTAR) models. van Dijk et al. (2002), among many others, discussed MSTAR models. Montgomery, et al. (1998) and Marcellino (2002) report much more favorable forecasting performance for LSTAR forecasts, while the results obtained in Stock and Watson (1999) show that linear models generally dominate nonlinear models in terms of forecasting performance. In spite of specification difficulties, such as the appropriate transition variable, number of regimes, type of transition function, and so on, they prove useful for state dependent multivariate relationships. Recent applications (e.g., Rothman *et al.*, 2001; Psaradakis *et al.*, 2005; Tsay, 1998; De Gooijer and Vidiella-i-Anguera, 2004) find that MSTAR models successfully model nonlinear economic time-series data.

Here, we discuss the specification of MSTAR models, which also follows for the (univariate) STAR models. Define  $y_t = (y_{1t}, y_{2t}, \dots, y_{nt})'$  as  $(k \times 1)$  vector time series. In our case,

$y = (TRS, GGR, LI, CI)'$ , where all variables are in logarithms. We specify the  $k$ -dimensional MSTAR model as follows:

$$(11) \quad \Delta y_t = (\Theta_{1,0} + \sum_{j=1}^p \Theta_{1,j} \Delta y_{t-j}) + (\Theta_{2,0} + \sum_{j=1}^p \Theta_{2,j} \Delta y_{t-j}) G(s_t; \gamma, c) + \varepsilon_t,$$

where  $\Delta$  denotes the first difference operator such that  $\Delta x_t = x_t - x_{t-1}$ ,  $\Theta_{i,0}$ ,  $i = 1, 2$ , are  $(k \times 1)$  vectors,  $\Theta_{i,j}$ ,  $i = 1, 2$ ,  $j = 1, 2, \dots, p$ , are  $(k \times k)$  matrices, and  $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t}, \dots, \varepsilon_{kt})$  is a  $k$ -dimensional vector of white noise processes with zero mean and nonsingular covariance matrix  $\Sigma$ ,  $G(\cdot)$  is the transition function that controls smooth moves between the two regimes, and  $s_t$  is the transition variable. In both univariate and multivariate cases, we allow the transition variable  $s_t$  to equal any lagged component of  $y_t$ .

The (M)STAR model in equation (11) defines for two regimes, one associated with  $G(s_t; \gamma, c) = 0$  and another associated with  $G(s_t; \gamma, c) = 1$ . The transition from one regime to the other is smooth and determined by the shape of the  $G(\cdot)$  function. In this paper, we consider a logistic transition function

$$(12) \quad G(s_t; \gamma, c) = \frac{1}{1 + \exp\{-\gamma(s_t - c)/\hat{\sigma}_s\}}, \quad \gamma > 0,$$

where  $\hat{\sigma}_s$  is the estimate of the standard deviation of transition variable  $s_t$ . The parameter  $c$  is the threshold determining the midpoint between two regimes at  $G(c; \gamma, c) = 0.5$ . The speed of transition between the regimes are determined by the parameter  $\gamma$ , with higher values corresponding to faster transition.

To specify both STAR and MSTAR models, we follow the procedure presented in Terasvirta (1998) (see also van Dijk, et al., 2002; Lundbergh and Terasvirta, 2002). The first step specifies the lag order of  $p = 3$ . We maintain this order in the univariate case as well.

The next step tests linearity against the MSTAR alternative. Since the MSTAR model contains parameters not identified under the alternative, we follow the approach of Luukkonen et al. (1988) and replace the transition function  $G(\cdot)$  with a suitable Taylor approximation to overcome the nuisance parameter problem. The testing procedure selects a logistic MSTAR model with a single threshold, which we maintain for the univariate case as well.

The third step in our MSTAR model identification selects the transition variable  $s_t$ . In order to identify the appropriate transition variable, we run the linearity tests for several candidates,  $s_{1t}, s_{2t}, \dots, s_{mt}$ , and select the one that gives the smallest  $p$ -value for the test statistic. Here, we consider lagged monthly changes of all four variables as the candidate transition variable. Let  $s_t = \Delta x_{t-d}$ , where  $x$  is any of the four variables  $\{TRS, GGR, LI, CI\}$ . We test linearity with these four variables for delays  $d = 1, 2, \dots, 8$ . We obtain the smallest  $p$ -value with  $x_t = TRS_t$  and  $d = 3$ . For the univariate case, we follow the same procedure to select  $s_t$ . We select  $TRS_{t-3}$  for taxable retail sales as the transition variable, and  $GGR_{t-2}$  for gross gaming revenue. Analytical point forecasts are not available for non-linear (V)AR models when the disturbance term is Gaussian even when  $h \geq 2$ , as  $E[f(x)] \neq f[E(x)]$ , where  $h$  is the number of steps-ahead for the forecasts.<sup>14</sup>

#### 4.4. Artificial Neural Network Model Identification

Artificial-neural-network (ANN) models perform well in forecasting nonlinear and chaotic time series (Lachtermacher and Fuller, 1995). As analogues to the STAR models, we consider both multivariate autoregressive ANN (MAR-ANN) and univariate autoregressive ANN (AR-ANN)

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<sup>14</sup> Details of the bootstrapping procedure are available upon request from the authors. We implement all computations of the STAR models with the RSTAR package (Version 0.1-1) in R developed by the one of the authors of this paper.



models. We estimate the univariate models only for forecasting taxable retail sales and gross gaming revenues. Lisi and Schiavo (1999) use an ANN models for predicting European exchange rates, finding that they performed as well as the best model, a chaos model. Using statistical tests, Lisi and Schiavo (1999) discover no significant difference between the ANN and chaos models. Stern (1996) applies ANN models to several simulated data from autoregressive models of order 2, AR(2), with various signal to noise ratios. The results showed that ANN models do not generate good predictions with a small signal to noise ratio. ANN models seem most suitable for forecasting time series with small signal to noise ratios, given sufficient data and appropriate data transformations. Success of ANNs in forecasting nonlinear time series reflects their universal function approximation capability. This includes any linear or nonlinear function (Cybenko, 1989; Funahashi, 1989; Hornik, et al., 1989; Wasserman, 1989). Because of this approximation capacity, neural networks offer several potential advantages over alternative methods for non-normal and non-linear data (Hansen et al., 1999).

Researchers use a variety of neural-net architectures for time-series prediction. The most widely used architecture for time series prediction is the multilayer perceptron (MLP) (also known as a feed-forward neural network) (Sarle, 2002). The MLP is capable of resolving a wide variety of problems (Bishop, 1995; Kaastra, Boyd, 1996). In this paper, we also prefer the MLP network for (M)AR-ANN based forecasting. In an MLP network, the units are partitioned into layers. Usually, the MLP network contains an input and an output layer, and one or more hidden layers of neurons between the input and output layers. In the MLP architecture, data are always transmitted from the input layer to the output layer. In our case, each input neuron represents one of the lagged values, while the output neuron(s) represent dependent variable(s) or MLP network forecasts. The MLP is a network with links from each unit in the  $k$ th layer directed only to units

in the  $(k + 1)$ st layer. In the (M)AR-ANN models, the lags of variables enter as inputs to the first layer, and outputs from the network appear in the last layer. A weight (“connection strength”) is associated with each link, and a network is trained (“learned”) by modifying these weights, thereby modifying the network function that maps inputs to outputs. We use the (M)AR-ANN model with  $q$ -hidden layers, which we write as follows:

$$(13) \quad \Delta y_t = \sum_{i=1}^q \beta'_i G(\Theta_{i,0} + \sum_{j=1}^p \Theta_{i,j} \Delta y_{t-j}) + \varepsilon_t,$$

where  $\Delta$  and  $y_t$  are as before,  $\beta_i$ ,  $i=1,2,\dots,q$ , are parameters called weights or connection strengths,  $\Theta_{i,0}$ ,  $i=1,2,\dots,q$ , are  $(k \times 1)$  vectors,  $\Theta_{i,j}$ ,  $i=1,2,\dots,q$ ,  $j=1,2,\dots,p$ , are  $(k \times k)$  matrices,  $G(\cdot)$  is the “squashing (activation) function” called the “hidden unit”, and  $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t}, \dots, \varepsilon_{kt})$  is a  $k$ -dimensional vector of white noise processes with zero mean and nonsingular covariance matrix  $\Sigma$ .

In building ANNs for forecasting time series, researchers frequently subdivide the sample into three sets (Bishop, 1995; Ripley, 1996). These sets are called training, validation, and test sets. The training set is used to construct the network, the validation set is used to obtain forecast performance measures, and the test set is used to check for generalization capacity of the network. This method can usefully construct networks with good generalization capability that performs well with new cases. During the network’s training stage, the weights iteratively adjust, using an algorithm such as the back propagation of Rumelhart, et al. (1986), on the basis of the training set’s values, in order to minimize the error between the network’s predicted output and the actual (desired) output. We use sum-of-squared errors (SSE) as a criterion to determine the optimal weights based on the training set. Nevertheless, ANN training based on the training set may lead to overfitting. In order to avoid overfitting, the validation set controls the learning

process. We evaluate an ANN's performance by changing the number of hidden layers and type of activation function at hidden and output layers, on the basis of the mean squared error (MSE) obtained when the trained ANN forecasts the period in the validation set. Finally, the test set, which is an independent set of data, provides an unbiased estimate of the generalization error or forecasting performance. No optimal rules exist to select the size of each set of data, although by general agreement, the training set should be the largest. In this paper, we use data from 1982:M2 to 2002:M12 as the training set (251 observations, 74.93%), data from 2003:M1 to 2006:M5 as the validation set (41 observations, 12.24%), and data from 2006:M6 to 2009:M12 as the test set (43 observations, 12.84%). We evaluate a network's performance based on the 1 to 24 step-ahead forecasts in the validation period and we select the best performing network based on the minimum MSE. Then, we select the network that based on the validation set is used to forecast the test period.<sup>15</sup>

Creating an MLP network involves five sets of parameters: the learning rule, network architecture, learning rate and momentum factor, activation function of the hidden and output layers, and number of iterations. Over the years, researchers develop many methods to train an ANN. (see Fine, 1999). MacKay (1992) proposed a Bayesian framework, called the Bayesian regularization, to overcome the problems in interpolation of noisy data. Bayesian regularization facilitates the selection of parsimonious models as well as maximum likelihood estimation. Bayesian regularization advantageously expands the cost function to search not only for the minimal error, but also for the minimal error using the minimal weights. In the Bayesian regularization approach, one determines a set of smaller models nested within a larger model and the algorithm chooses one of these smaller models, providing a method to select parsimonious

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<sup>15</sup> See Section 5 for further details.

models. The procedure first assigns prior probabilities to each of the smaller models and then determines the model that posts the highest posterior probability. Following the recommendation in Foresee and Hagan (1997), we fit the models using the Levenberg–Marquardt algorithm.

In this paper, the MLP architecture uses three lags of each variable as inputs for MAR-ANN and three lags of own for AR-ANN models of taxable retail sales and gross gaming revenues. An MLP network's capacity to learn depends on the number of hidden neurons. Despite its significant role, no statistical criteria exist to select the optimum number of hidden neurons. We select the best ANN with Bayesian regularization, bearing in mind the overfitting issue, based on its MSE in the validation set, using the least possible number of hidden neurons (Masters, 1993; Smith, 1993; Rzepoluck, 1998). For both MAR-ANN and AR-ANN models, we try ANNs with maximum  $q$  set to 9. We obtain the best performing MAR-ANN with  $q = 3$ , and the best performing AR-ANN with  $q = 2$  for taxable retail sales and with  $q = 1$  for gross gaming revenue.

In our study, the input layer neurons use a linear activation function, while the hidden and output layer neurons use a sigmoid activation function,  $G(\cdot)$ . Two sigmoid functions widely used in MLP are the logistic (providing continuous values between 0 and 1) and hyperbolic tangent sigmoid, called tansig, functions (providing continuous values between -1 and 1). In this study, we use the tansig function in the hidden and output layers of the MLP networks, since it allows much faster learning in comparison to the logistic function (Fahlman, 1988; Fausett, 1994). We scale our data onto -1 and 1, which is the range covered by the tansig function.

The learning rate parameter plays a crucial role in the training process of MLP networks. The learning rate controls the change in the weights in each iteration of training. In order to obtain optimum weights, researchers should avoid both too-small and too-large size changes in

weights. We use a learning rate of 0.25, which provides good results in most practical cases (Rumelhart et al., 1986). We can increase the speed of learning by filtering, based on the past changes, the oscillations caused by the learning rate. The momentum factor parameter controls the effect of past changes, which should be a number close to 1. In this study, we use a momentum factor equal to 0.85.<sup>16</sup>

## **5. Data and Results:**

This section reports our data sources and econometric findings. In addition to the monthly Nevada coincident and leading employment indexes, we use monthly data from January 1982 through December 2009 on seasonally-adjusted Nevada gross gaming revenue and taxable retail sales. We use January 1982 through December 2002 as the in-sample period and January 2003 through May 2006 as the out-of-sample period, which is updated recursively to generate one- to twenty four-months-ahead forecasts. Our forecasting analysis compares the various models against a benchmark random-walk model. Finally we use the period from June 2006 to December 2009 for carrying out ex ante forecasts in an attempt to predict the downturn in gross gaming revenue (which peaked in November 2006) and taxable retail sales (which peaked in February 2007).

### *5.1. VAR, VEC, BVAR, and BVEC Forecast Results*

We begin with a series of linear forecasting models. The best performing models bifurcate across taxable sales and gross gaming revenue. For taxable sales, the VEC models generally provide the best forecasting performance on average across all forecast horizons as well as at each individual forecast horizon from 1 to 24 months. Occasionally, the BVEC models outperform the VEC models, but by a small margin. Moreover, the BVEC models generally rank second in forecast

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<sup>16</sup> We implement all computations of the ANN models with the Neural Network Toolbox (Version 6.0) in MATLAB.

performance, usually by a small margin. In sum, the VEC and BVEC models provide the best forecasting performance for taxable sales with average performance about 44 percent better than the random walk model benchmark.

For gross gaming revenue, the VAR models generally provide the best forecasting performance on average across all forecast horizons as well as at each individual forecast horizon from 1 to 24 months. The BVAR models outperform the VAR models at the first three months forecast horizon. Moreover, occasionally, the VEC or BVEC model outperforms the VAR model. The VEC and BVEC models, however, show an erratic forecasting performance with excellent performance at some horizons and awful performance at other horizons. The VAR and BVAR models show a consistent pattern across all forecast horizons and both forecasts' RMSEs differ only marginally. In sum, the VAR and BVAR models provide the best forecasting performance for taxable sales with average performance about 34 percent better than the random walk model benchmark.

Figures 3 and 4 provide out-of-sample ex ante forecasts of taxable sales and gross gaming revenue from June 2006 through December 2009. The linear nature of the optimal VEC and VAR models used to forecast taxable sales and gross gaming revenue make it difficult to move with the twists and turns in either series over the sample period, which saw significant ups and downs in these components to Nevada's tax base. Figures 5 and 6 present the ex ante forecasts for the random-walk model for comparison purposes.

## *5.2. Nonparametric and Semi-Parametric Forecast Results*

The nonparametric and semi-parametric forecasting models generally perform poorly. On average, all three models perform worse than the random walk model. For example, Model 1 produced an especially bad forecast performance for gross gaming revenue with an average of

almost 2000 percent worse than the random-walk model. The performance of these three models improves somewhat for longer forecast horizons, especially for the semi-parametric model. The semi-parametric model outperforms the random-walk model from forecast horizon 11 and 13 through 24 for taxable sales and gross gaming revenue, respectively.

Figures 7 and 8 provide out-of-sample ex ante forecasts of taxable sales and gross gaming revenue from June 2006 through December 2009. The optimal semi-parametric model does a much better job than the optimal VEC and VAR models in the prior subsection in keeping the forecast values near the actual values. This outcome occurs in spite of the fact that the semi-parametric model did not outperform the random walk model.

### *5.3. Smooth Transition Autoregressive Forecast Results*

The smooth transition autoregressive forecasts prove much better at forecasting taxable sales and gross gaming revenue than the prior techniques. Generally, the STAR models outperform the MSTAR models, although the differences in performance are not large. From forecast horizon 4 to 24, the STAR model dominates. For horizons 1 to 3, we flip flop between the STAR and MSTAR model as the best performing model. On average, the STAR model outperforms the random-walk model by about 85 percent.

Figures 9 and 10 provide out-of-sample ex ante forecasts of taxable sales and gross gaming revenue from June 2006 through December 2009. For both variables, the ex ante forecasts quickly evolve into a no-change forecast over most of the out-of-sample forecast period.

### *5.4. Artificial Neural Network Forecast Results*

The autoregressive artificial neural network model outperforms (or equals in one case) the multivariate autoregressive neural network model at all forecast horizons. Moreover, as we noted

for the smooth transition autoregressive models, the performance of the AR-ANN model beats the random-walk model by a large amount. Comparing the two models, the autoregressive artificial neural network model outperforms the smooth transition autoregressive model by a small amount in 20 and 21 out of 24 forecast horizons for taxable sales and gross gaming revenue, respectively. On average, the AR-ANN model beats the random-walk model by about 85 to 86 percent.

Figures 11 and 12 provide out-of-sample ex ante forecasts of taxable sales and gross gaming revenue from June 2006 through December 2009. These graphs tell a story similar to the ex ante forecasts using the smooth transition autoregressive models reported in Figures 9 and 10.

## **6. Conclusions:**

Most state governments face some constitutional or legislative requirement to balance their current services budget. Nevada is no exception. Thus, states necessarily need to forecast revenue in order to determine the level of government spending that the forecast revenue can support. In Nevada, the Economic Forum, a group of five laypersons, makes revenue forecasts that bind the government to spending limits. The Economic Forum hears testimony from various constituencies, including the legislative and executive branches of government and attempts to craft a consensus forecast. Taxable sales and gross gaming revenue comprises a significant portion of Nevada's tax base.

Table 6 summarizes much of our findings. This Table reports the forecast performance of the autoregressive artificial-neural-network models versus the random-walk model and then between the autoregressive artificial-neural-network models and, in turn, the semi-parametric models, the vector-error-correction and vector-autoregressive models for taxable sales and gaming revenue, respectively, and the smooth-transition autoregressive models. That is, the



autoregressive artificial neural network and smooth transition autoregressive models provide the best performance of all of the models by a significant margin, but we do not see statistical differences between the forecast errors of artificial-neural-network and smooth-transition autoregressive models, based on the Diebold and Mariano (1995) test statistic.<sup>17</sup>

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<sup>17</sup> See Appendix for more explanation.

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**Table 1: VAR, BVAR, VEC, and BVEC Forecast Results: Taxable Sales**

Forecast Horizon				w=0.3, d=0.5			w=0.2, d=1			w=0.1, d=1			w=0.2, d=2			w=0.1, d=2		
	AR	VAR	VEC	UBVAR	BVAR	BVEC	UBVAR	BVAR	BVEC	UBVAR	BVAR	BVEC	UBVAR	BVAR	BVEC	UBVAR	BVAR	BVEC
1	0.92	1.30	<b>0.77</b>	0.91	1.29	<b>0.77</b>	0.92	1.34	<b>0.77</b>	0.94	1.33	<b>0.77</b>	0.95	1.48	<b>0.77</b>	0.98	1.43	<b>0.77</b>
2	0.89	1.32	<b>0.48</b>	0.88	1.35	0.48	0.89	1.49	0.48	0.91	1.58	0.50	0.92	1.77	0.50	0.96	1.78	0.52
3	0.92	1.43	<b>0.33</b>	0.90	1.50	0.33	0.91	1.72	0.33	0.92	1.89	0.33	0.93	2.13	0.34	0.95	2.19	0.36
4	0.86	1.38	<b>0.41</b>	0.84	1.45	0.41	0.85	1.66	0.41	0.88	1.84	0.41	0.90	2.04	0.42	0.93	2.12	0.44
5	0.86	1.40	<b>0.50</b>	0.83	1.49	0.52	0.84	1.73	0.53	0.86	1.93	0.53	0.88	2.13	0.54	0.91	2.23	0.56
6	0.85	1.41	<b>0.40</b>	0.82	1.50	0.43	0.83	1.75	0.43	0.85	1.96	0.44	0.87	2.14	0.44	0.90	2.24	0.47
7	0.84	1.35	<b>0.53</b>	0.81	1.44	0.57	0.82	1.67	0.57	0.84	1.86	0.58	0.86	2.01	0.58	0.90	2.10	0.61
8	0.85	1.34	<b>0.57</b>	0.81	1.44	0.61	0.82	1.67	0.61	0.84	1.85	0.61	0.86	1.99	0.62	0.90	2.07	0.64
9	0.84	1.30	<b>0.49</b>	0.81	1.39	0.56	0.82	1.61	0.56	0.84	1.78	0.56	0.86	1.90	0.57	0.90	1.97	0.60
10	0.83	1.26	<b>0.55</b>	0.80	1.35	0.59	0.81	1.55	0.59	0.84	1.71	0.60	0.86	1.81	0.60	0.90	1.87	0.63
11	0.82	1.24	<b>0.47</b>	0.79	1.33	0.55	0.81	1.53	0.56	0.83	1.68	0.56	0.86	1.77	0.57	0.90	1.83	0.60
12	0.81	1.21	<b>0.48</b>	0.78	1.30	0.53	0.80	1.49	0.53	0.83	1.63	0.54	0.85	1.71	0.54	0.90	1.76	0.58
13	0.81	1.20	<b>0.58</b>	0.78	1.29	0.66	0.79	1.47	0.67	0.83	1.60	0.67	0.85	1.67	0.67	0.89	1.72	0.70
14	0.82	1.20	<b>0.58</b>	0.78	1.29	0.61	0.80	1.47	0.62	0.82	1.59	0.62	0.85	1.66	0.63	0.89	1.70	0.65
15	0.81	1.17	<b>0.61</b>	0.78	1.26	0.70	0.79	1.43	0.70	0.83	1.54	0.71	0.85	1.60	0.71	0.89	1.63	0.73
16	0.81	1.16	<b>0.63</b>	0.78	1.24	0.65	0.79	1.40	0.65	0.83	1.51	0.65	0.85	1.56	0.66	0.89	1.59	0.68
17	0.81	1.16	<b>0.57</b>	0.78	1.24	0.69	0.79	1.40	0.69	0.82	1.50	0.70	0.85	1.55	0.70	0.89	1.57	0.73
18	0.81	1.14	0.61	0.77	1.22	<b>0.60</b>	0.79	1.37	<b>0.60</b>	0.82	1.46	0.61	0.85	1.50	0.61	0.89	1.53	0.63
19	0.81	1.14	<b>0.61</b>	0.77	1.21	0.75	0.79	1.35	0.75	0.82	1.44	0.76	0.85	1.48	0.76	0.89	1.50	0.78
20	0.81	1.14	0.60	0.77	1.21	<b>0.54</b>	0.79	1.35	<b>0.54</b>	0.82	1.43	0.55	0.85	1.46	0.55	0.89	1.48	0.58
21	0.80	1.13	<b>0.64</b>	0.77	1.20	0.82	0.79	1.32	0.82	0.82	1.40	0.83	0.85	1.43	0.82	0.89	1.44	0.84
22	0.80	1.13	0.63	0.77	1.19	<b>0.51</b>	0.79	1.32	0.52	0.82	1.39	0.52	0.85	1.42	0.53	0.89	1.43	0.56
23	0.80	1.13	<b>0.63</b>	0.77	1.19	0.89	0.79	1.31	0.90	0.82	1.37	0.90	0.85	1.40	0.90	0.89	1.41	0.91
24	0.80	1.12	0.65	0.76	1.18	<b>0.43</b>	0.78	1.29	0.44	0.82	1.35	0.44	0.85	1.37	0.45	0.90	1.38	0.49
Mean	0.83	1.24	<b>0.56</b>	0.80	1.31	0.59	0.82	1.49	0.59	0.85	1.61	0.60	0.87	1.71	0.60	0.91	1.75	0.63

**Note:** Numbers represent the ratio of the root mean squared error (RMSE) of the model relative to the RMSE of the random-walk (RW) model. Thus, a ratio less (more) than one implies that the model in question performs better (worse) than the RW model in forecasting at a particular forecast horizon. AR, VAR, and VEC mean autoregressive, vector autoregressive, and vector error-correction models. BVAR, UBVAR, and BVEC mean Bayesian VAR, univariate BVAR, and Bayesian VEC models. Shaded cells with bold numbers represent the best performing model at forecasting at each forecast horizon. Ave calculates the averages of the 1 to 24 month forecast horizons for each model. For the BVAR and BVEC models,  $w$  and  $d$  represent the tightness values chosen for the hyper parameters in the Bayesian specification of the standard errors of the priors on the parameters in the Minnesota prior. We adopt the standard specification for the weighting matrix.

**Table 2: VAR, BVAR, VEC, and BVEC Forecast Results: Gross Gaming Revenue**

Forecast Horizon				w=0.3, d=0.5			w=0.2,d=1			w=0.1,d=1			w=0.2, d=2			w=0.1,d=2		
	AR	VAR	VEC	UBVAR	BVAR	BVEC	UBVAR	BVAR	BVEC	UBVAR	BVAR	BVEC	UBVAR	BVAR	BVEC	UBVAR	BVAR	BVEC
1	0.80	0.80	0.81	0.81	<b>0.79</b>	0.81	0.82	0.80	0.81	0.87	0.83	0.81	0.89	0.83	0.81	0.96	0.86	0.81
2	0.81	0.79	1.40	0.82	0.79	1.40	0.83	<b>0.78</b>	1.40	0.87	0.80	1.40	0.89	0.81	1.39	0.96	0.83	1.37
3	0.87	0.85	2.07	0.88	0.84	2.07	0.89	<b>0.84</b>	2.07	0.92	0.84	2.05	0.93	0.86	2.04	0.99	0.85	1.98
4	0.81	<b>0.75</b>	2.39	0.83	0.75	2.39	0.85	0.78	2.39	0.90	0.81	2.37	0.92	0.82	2.36	0.99	0.82	2.29
5	0.80	<b>0.71</b>	2.17	0.82	0.72	2.23	0.84	0.75	2.23	0.90	0.79	2.21	0.93	0.81	2.19	1.01	0.82	2.14
6	0.81	<b>0.70</b>	2.17	0.83	0.71	2.34	0.85	0.76	2.31	0.90	0.80	2.29	0.94	0.83	2.26	1.02	0.84	2.11
7	0.84	<b>0.72</b>	16.14	0.86	0.74	17.86	0.88	0.78	17.86	0.92	0.82	17.71	0.96	0.86	17.43	1.04	0.86	16.57
8	0.87	<b>0.74</b>	4.12	0.89	0.75	4.39	0.90	0.79	4.39	0.94	0.82	4.34	0.98	0.85	4.32	1.05	0.85	4.17
9	0.84	<b>0.68</b>	5.70	0.87	0.70	6.70	0.89	0.74	6.70	0.94	0.77	6.60	0.98	0.79	6.50	1.06	0.78	6.15
10	0.82	<b>0.64</b>	6.60	0.85	0.66	7.20	0.87	0.70	7.20	0.93	0.73	7.12	0.97	0.75	7.08	1.06	0.74	6.80
11	0.82	<b>0.64</b>	6.80	0.85	0.67	7.88	0.88	0.72	7.88	0.93	0.75	7.80	0.98	0.77	7.72	1.07	0.76	7.40
12	0.81	<b>0.60</b>	4.47	0.84	0.63	4.86	0.87	0.69	4.86	0.93	0.73	4.81	0.98	0.75	4.79	1.08	0.73	4.63
13	0.79	<b>0.57</b>	5.42	0.83	0.61	6.88	0.86	0.67	6.83	0.93	0.70	6.75	0.97	0.72	6.67	1.08	0.71	6.29
14	0.81	0.62	<b>0.51</b>	0.84	0.66	0.66	0.87	0.72	0.65	0.94	0.74	0.63	0.98	0.77	0.63	1.09	0.75	0.56
15	0.80	<b>0.59</b>	1.06	0.83	0.62	1.64	0.87	0.68	1.63	0.94	0.71	1.60	0.99	0.73	1.56	1.09	0.70	1.45
16	0.80	<b>0.58</b>	1.69	0.83	0.61	1.84	0.86	0.67	1.82	0.93	0.70	1.80	0.98	0.71	1.79	1.09	0.70	1.67
17	0.81	<b>0.61</b>	3.31	0.84	0.64	4.79	0.88	0.70	4.77	0.94	0.72	4.72	0.99	0.74	4.64	1.10	0.72	4.36
18	0.80	<b>0.58</b>	1.46	0.83	0.62	1.39	0.87	0.67	1.39	0.94	0.70	1.36	0.99	0.71	1.36	1.10	0.69	1.27
19	0.79	<b>0.58</b>	3.29	0.82	0.62	4.31	0.86	0.68	4.30	0.93	0.70	4.26	0.99	0.71	4.21	1.10	0.69	4.03
20	0.80	<b>0.60</b>	1.36	0.83	0.64	0.96	0.87	0.69	0.96	0.93	0.71	0.95	0.99	0.72	0.95	1.10	0.70	0.91
21	0.80	<b>0.61</b>	0.87	0.83	0.64	2.04	0.87	0.69	2.03	0.94	0.71	1.99	0.99	0.72	1.93	1.10	0.70	1.76
22	0.78	0.60	1.02	0.82	0.63	0.21	0.86	0.69	<b>0.21</b>	0.93	0.70	0.21	0.99	0.71	0.24	1.10	0.69	0.25
23	0.79	<b>0.63</b>	1.12	0.82	0.66	3.06	0.86	0.71	3.04	0.93	0.73	2.99	0.98	0.73	2.89	1.10	0.71	2.62
24	0.80	0.64	3.97	0.83	0.67	<b>0.05</b>	0.87	0.71	0.08	0.94	0.72	0.10	0.99	0.73	0.28	1.10	0.70	0.51
Mean	0.81	0.66	3.33	0.84	0.68	3.66	0.87	0.73	3.66	0.92	0.75	3.62	0.97	0.77	3.58	1.06	0.76	3.42

**Note:** See Table 1.

**Table 3: Non-Parametric and Semi-Parametric Forecast Results**

<b>Forecast Horizon</b>	<b>Model 1: Non-Parametric</b>		<b>Model 2: Non-Parametric</b>		<b>Model 3: Semi-Parametric</b>	
	<b>Taxable Sales</b>	<b>Gaming Revenue</b>	<b>Taxable Sales</b>	<b>Gaming Revenue</b>	<b>Taxable Sales</b>	<b>Gaming Revenue</b>
<b>1</b>	4.555	2.100	4.429	<b>1.492</b>	<b>4.426</b>	1.773
<b>2</b>	3.652	2.276	3.418	<b>1.682</b>	<b>3.353</b>	1.710
<b>3</b>	3.578	2.094	3.053	<b>1.956</b>	<b>2.841</b>	2.156
<b>4</b>	2.883	1.931	2.569	1.723	<b>2.420</b>	<b>1.619</b>
<b>5</b>	2.851	2.229	2.509	1.774	<b>2.216</b>	<b>1.551</b>
<b>6</b>	2.689	2.500	2.316	1.822	<b>2.018</b>	<b>1.657</b>
<b>7</b>	2.369	2.796	2.065	1.871	<b>1.798</b>	<b>1.536</b>
<b>8</b>	2.219	2.743	1.879	1.745	<b>1.568</b>	<b>1.476</b>
<b>9</b>	1.962	2.350	1.663	1.382	<b>1.360</b>	<b>1.189</b>
<b>10</b>	1.802	2.201	1.527	1.307	<b>1.160</b>	<b>1.023</b>
<b>11</b>	1.649	2.289	1.418	1.344	<b>0.963</b>	<b>1.051</b>
<b>12</b>	1.484	2.333	1.363	1.283	<b>0.898</b>	<b>1.070</b>
<b>13</b>	1.370	2.011	1.362	1.235	<b>0.848</b>	<b>0.941</b>
<b>14</b>	1.203	2.239	1.324	1.277	<b>0.854</b>	<b>0.951</b>
<b>15</b>	1.061	51.252	1.242	1.186	<b>0.818</b>	<b>0.937</b>
<b>16</b>	0.967	49.756	1.170	1.135	<b>0.686</b>	<b>0.803</b>
<b>17</b>	0.931	49.582	1.143	1.094	<b>0.642</b>	<b>0.674</b>
<b>18</b>	0.892	46.818	1.116	1.042	<b>0.617</b>	<b>0.678</b>
<b>19</b>	0.838	46.377	1.091	1.080	<b>0.574</b>	<b>0.738</b>
<b>20</b>	0.777	45.326	1.066	1.088	<b>0.605</b>	<b>0.692</b>
<b>21</b>	0.724	44.029	1.041	1.008	<b>0.594</b>	<b>0.674</b>
<b>22</b>	0.668	42.788	1.030	0.996	<b>0.518</b>	<b>0.550</b>
<b>23</b>	0.581	43.724	1.011	1.046	<b>0.500</b>	<b>0.658</b>
<b>24</b>	0.542	42.454	0.985	1.001	<b>0.515</b>	<b>0.597</b>
<b>Average</b>	1.760	20.592	1.741	1.357	<b>1.366</b>	<b>1.113</b>

**Note:** See Table 1.

**Table 4: Smooth Transition Autoregressive Forecast Results**

Forecast Horizon	STAR		MSTAR	
	Taxable Sales	Gaming Revenue	Taxable Sales	Gaming Revenue
1	<b>0.102</b>	0.130	0.108	<b>0.128</b>
2	<b>0.170</b>	<b>0.234</b>	<b>0.170</b>	0.240
3	0.194	<b>0.236</b>	<b>0.183</b>	0.323
4	<b>0.128</b>	<b>0.198</b>	0.143	0.271
5	<b>0.148</b>	<b>0.212</b>	0.185	0.317
6	<b>0.165</b>	<b>0.215</b>	0.179	0.367
7	<b>0.135</b>	<b>0.208</b>	0.167	0.407
8	<b>0.143</b>	<b>0.180</b>	0.197	0.381
9	<b>0.135</b>	<b>0.179</b>	0.160	0.397
10	<b>0.127</b>	<b>0.152</b>	0.171	0.366
11	<b>0.134</b>	<b>0.151</b>	0.170	0.395
12	<b>0.135</b>	<b>0.157</b>	0.151	0.410
13	<b>0.134</b>	<b>0.131</b>	0.182	0.369
14	<b>0.136</b>	<b>0.147</b>	0.175	0.433
15	<b>0.133</b>	<b>0.124</b>	0.166	0.418
16	<b>0.140</b>	<b>0.123</b>	0.188	0.390
17	<b>0.143</b>	<b>0.144</b>	0.169	0.453
18	<b>0.141</b>	<b>0.114</b>	0.160	0.418
19	<b>0.144</b>	<b>0.114</b>	0.191	0.414
20	<b>0.145</b>	<b>0.128</b>	0.182	0.448
21	<b>0.145</b>	<b>0.112</b>	0.175	0.424
22	<b>0.148</b>	<b>0.099</b>	0.186	0.419
23	<b>0.149</b>	<b>0.130</b>	0.180	0.479
24	<b>0.149</b>	<b>0.106</b>	0.175	0.392
Average	<b>0.143</b>	<b>0.155</b>	0.171	0.377

**Note:** See Table 1. STAR and MSTAR mean smooth transition autoregressive and multivariate STAR models.



**Table 5: Artificial Neural Network Forecast Results**

Forecast Horizon	AR-ANN		MAR-ANN	
	Taxable Sales	Gaming Revenue	Taxable Sales	Gaming Revenue
1	0.127	0.128	0.130	0.128
2	0.165	0.237	0.181	0.240
3	0.190	0.236	0.209	0.323
4	0.130	0.192	0.153	0.271
5	0.146	0.208	0.190	0.317
6	0.163	0.214	0.185	0.367
7	0.135	0.203	0.168	0.406
8	0.142	0.175	0.187	0.381
9	0.135	0.178	0.166	0.397
10	0.127	0.152	0.181	0.366
11	0.134	0.148	0.199	0.394
12	0.135	0.154	0.198	0.410
13	0.133	0.126	0.230	0.369
14	0.135	0.143	0.223	0.433
15	0.132	0.120	0.196	0.417
16	0.138	0.119	0.266	0.389
17	0.142	0.139	0.219	0.453
18	0.139	0.109	0.219	0.417
19	0.142	0.110	0.255	0.413
20	0.143	0.124	0.220	0.448
21	0.142	0.110	0.272	0.424
22	0.145	0.097	0.311	0.418
23	0.147	0.128	0.205	0.479
24	0.146	0.104	0.224	0.391
Average	0.142	0.152	0.208	0.377

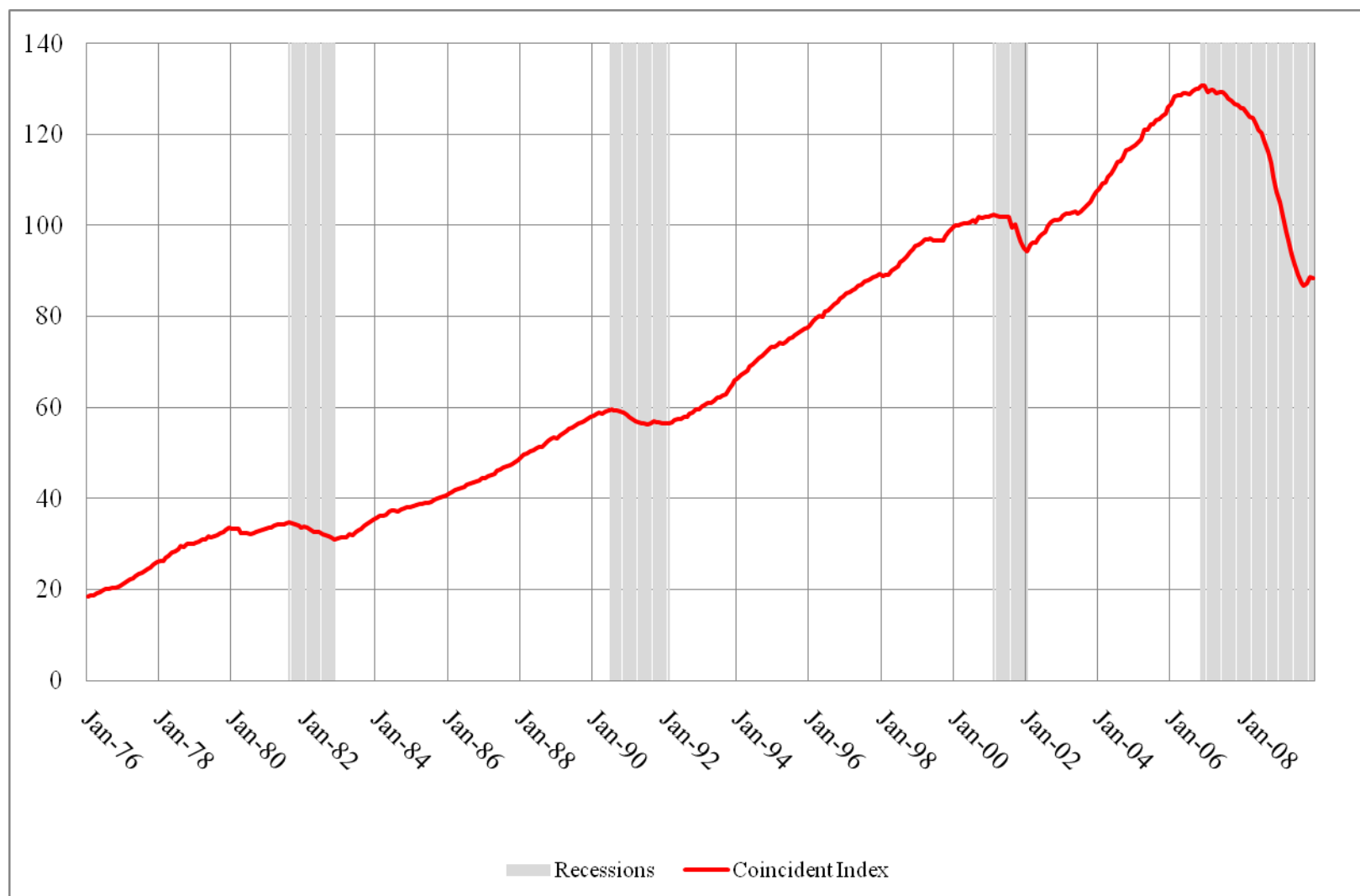
**Note:** See Table 1. AR-ANN and MAR-ANN mean autoregressive artificial neural network and multivariate AR-ANNN models.

**Table 6: Comparison AR-ANN Performance to Other Models**

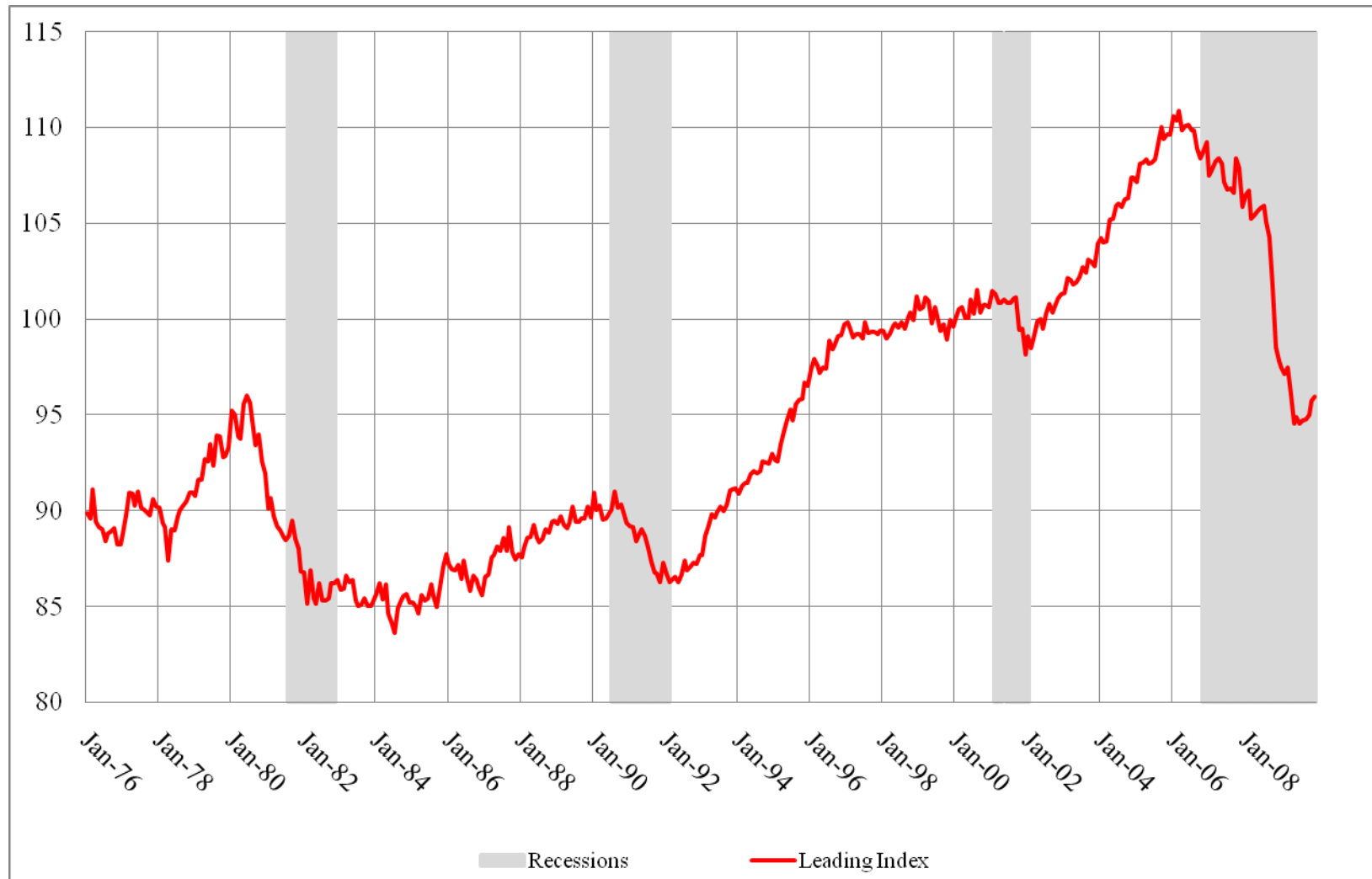
Forecast Horizon	AR-ANN		AR-ANN/SP		AR-ANN/VEC	AR-ANN/VAR	AR-ANN/STAR	
	Taxable Sales	Gaming Revenue	Taxable Sales	Gaming Revenue	Taxable Sales	Gaming Revenue	Taxable Sales	Gaming Revenue
1	0.127***	0.128***	0.029***	0.072***	0.166***	0.160***	1.245*	0.985
2	0.165***	0.237***	0.049***	0.139***	0.342***	0.299***	0.971	1.013
3	0.190***	0.236***	0.067***	0.109***	0.578**	0.278***	0.979	1.000
4	0.130***	0.192***	0.054***	0.119***	0.317***	0.256***	1.016	0.970
5	0.146***	0.208***	0.066***	0.134***	0.290***	0.294***	0.986	0.981
6	0.163**	0.214***	0.081***	0.129***	0.410***	0.308***	0.988	0.995
7	0.135***	0.203***	0.075***	0.132***	0.255***	0.281***	1.000	0.976
8	0.142***	0.175***	0.090***	0.119***	0.249***	0.237***	0.993	0.972
9	0.135***	0.178***	0.099***	0.150***	0.274***	0.264***	1.000	0.994
10	0.127***	0.152***	0.110***	0.149***	0.233***	0.238***	1.000	1.000
11	0.134***	0.148***	0.139***	0.141***	0.286***	0.231***	1.000	0.980
12	0.135***	0.154***	0.150***	0.144***	0.282***	0.256***	1.000	0.981
13	0.133***	0.126***	0.157***	0.134***	0.228***	0.221***	0.993	0.962
14	0.135***	0.143***	0.158***	0.150***	0.232***	0.230***	0.993	0.973
15	0.132***	0.120***	0.161***	0.128***	0.214***	0.204***	0.992	0.968
16	0.138***	0.119***	0.201***	0.148***	0.218***	0.204***	0.986	0.967
17	0.142***	0.139***	0.221***	0.207***	0.246***	0.229***	0.993	0.965
18	0.139***	0.109***	0.225***	0.161***	0.230***	0.187***	0.986	0.956
19	0.142***	0.110***	0.248***	0.149***	0.233***	0.189***	0.986	0.965
20	0.143***	0.124***	0.236***	0.180***	0.240***	0.206***	0.986	0.969
21	0.142***	0.110***	0.240***	0.163***	0.221***	0.182***	0.979	0.982
22	0.145***	0.097***	0.280***	0.175***	0.230***	0.161***	0.980	0.980
23	0.147***	0.128***	0.293***	0.195***	0.233***	0.203***	0.987	0.985
24	0.146***	0.104***	0.284***	0.175***	0.225***	0.164***	0.980	0.981
Average	0.142	0.152	0.104	0.137	0.256	0.231	1.001	0.979

**Note:** The columns headed by AR-ANN measures the RMSE for the AR-ANN relative to the RW model. The columns headed by AR-ANN/SP measures the RMSE for the AR-ANN model to the SP models in Table 3. The column headed by AR-ANN/VEC measures the RMSE for the AR-ANN model to the VEC model in Table 1. The column headed by AR-ANN/VAR measures the RMSE for the AR-ANN model relative to the VAR model in Table 2. The column headed by AR-ANN/STAR measures the RMSE for the AR-ANN model relative to the STAR model in Table 4. Finally, \*(\*\*)[\*\*\*] indicates 10%, (5%), [1%] level of Significance for the Diebold and Mariano (1995) tests statistic.

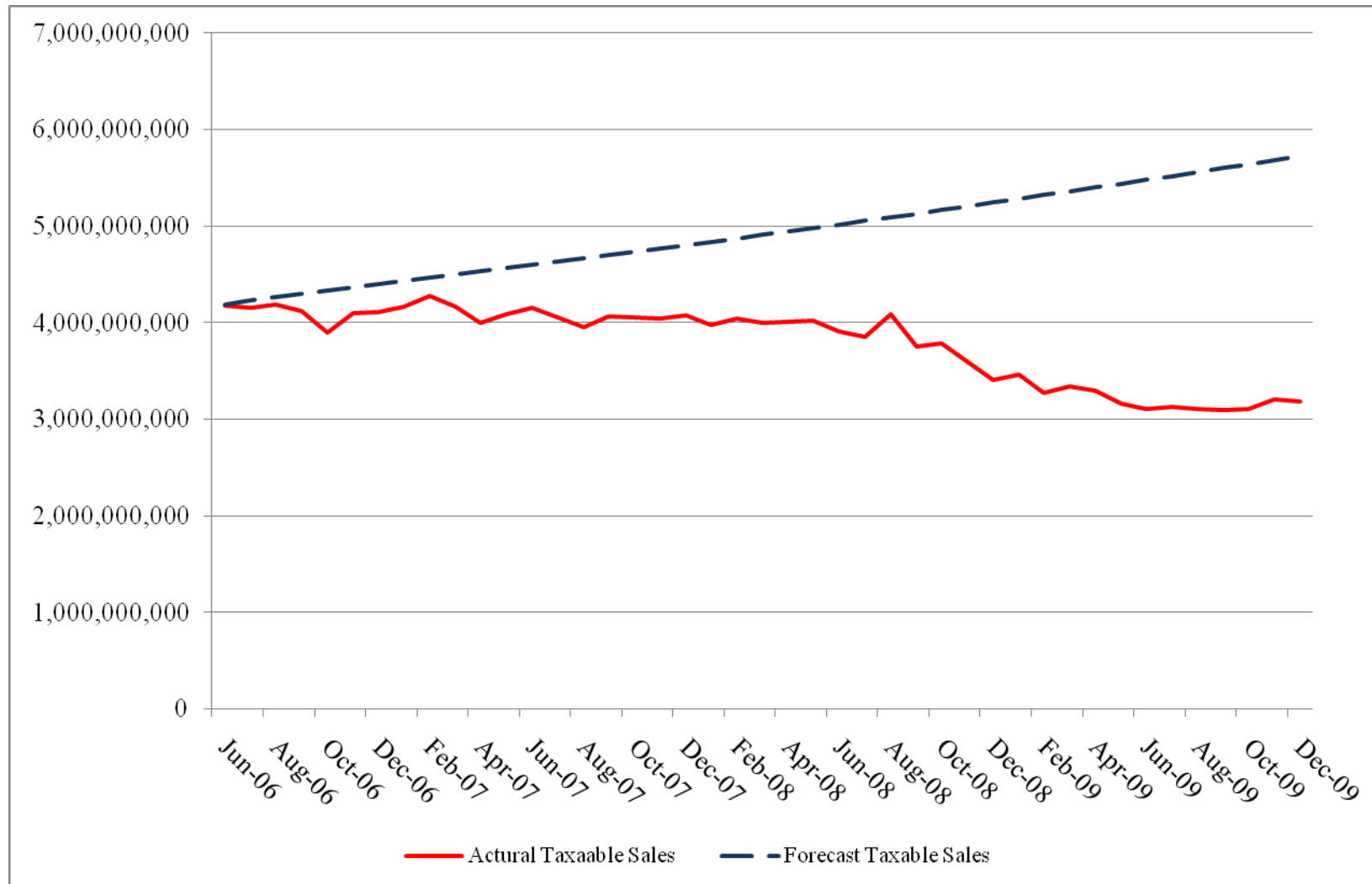
**Figure 1:      CBER-DETR Nevada Coincident Employment Index**



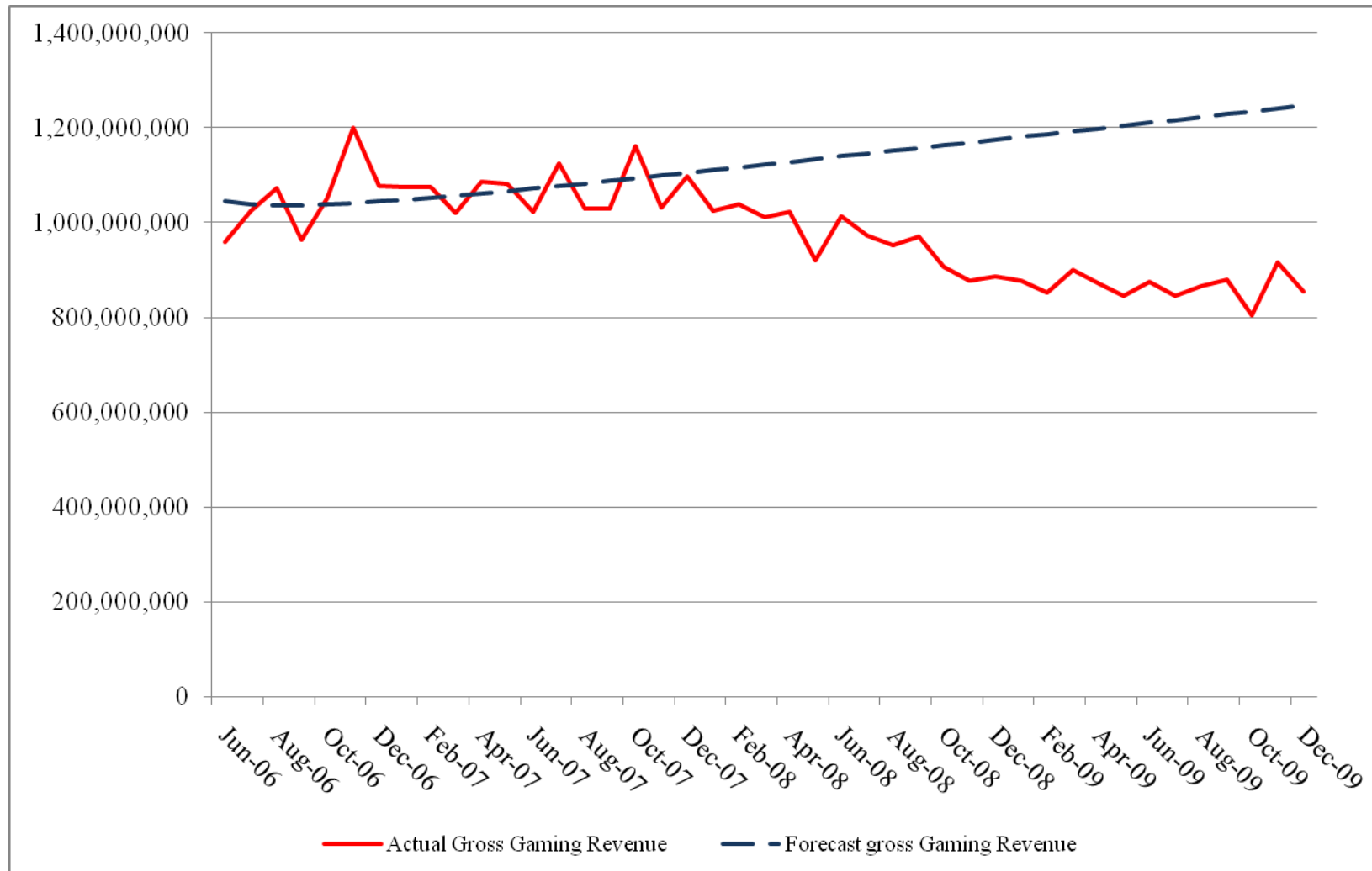
**Figure 2: CBER-DETR Nevada Leading Employment Index**



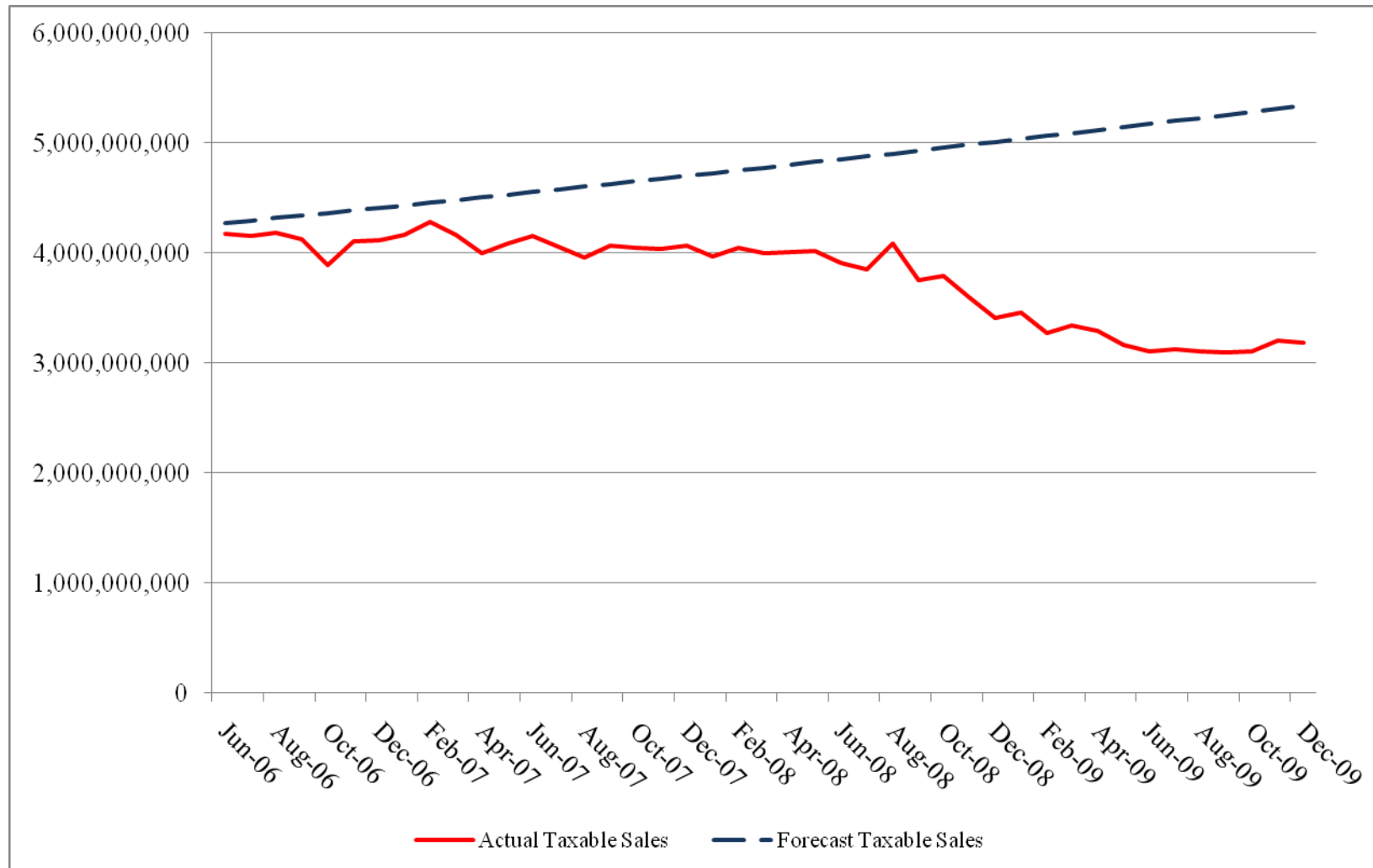
**Figure 3: Forecast and Actual Taxable Sales: Optimal VEC Model**



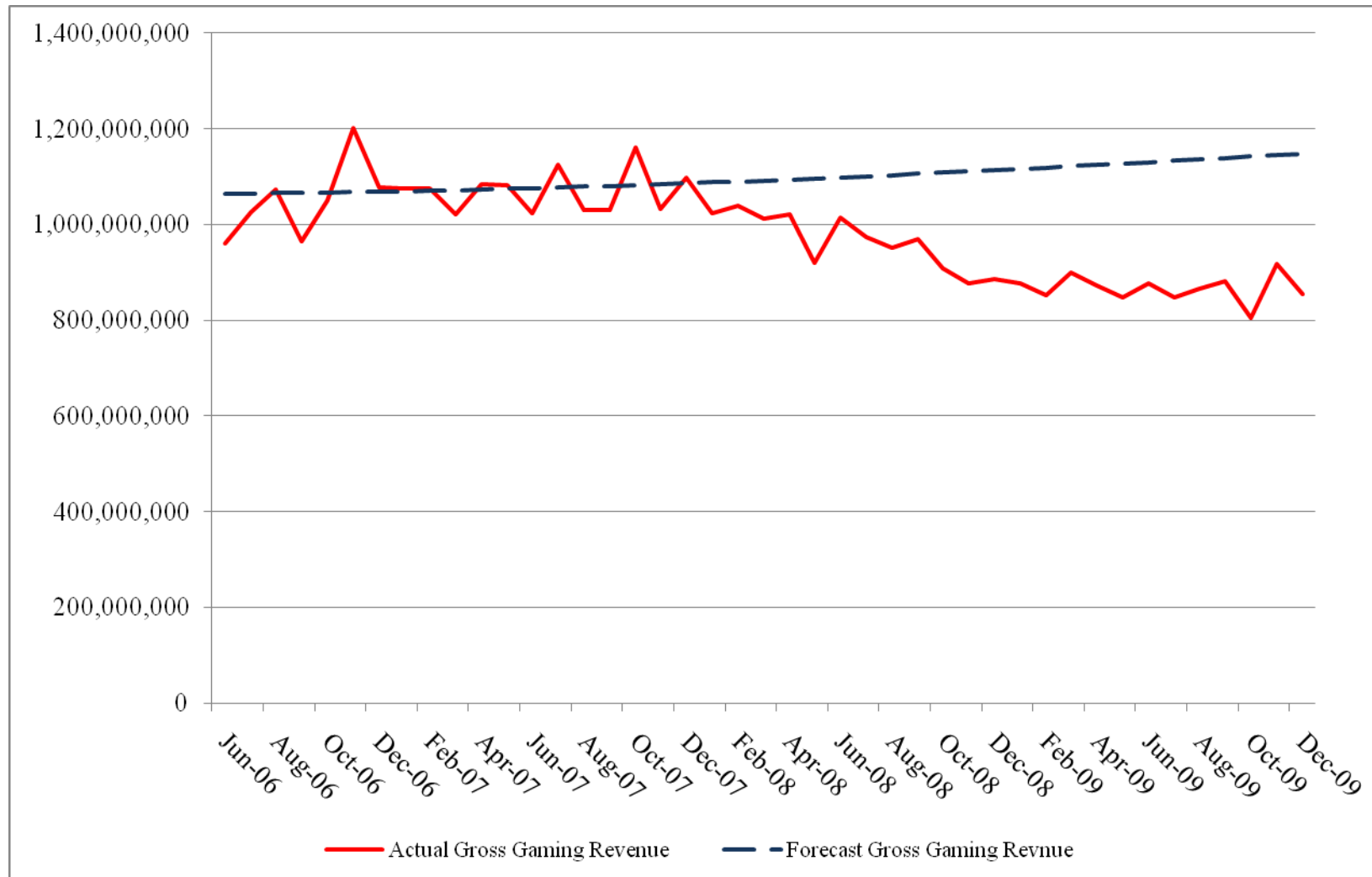
**Figure 4: Forecast and Actual Gross Gaming Revenue: Optimal VAR Model**



**Figure 5: Forecast and Actual Taxable Sales: Random-Walk Model**

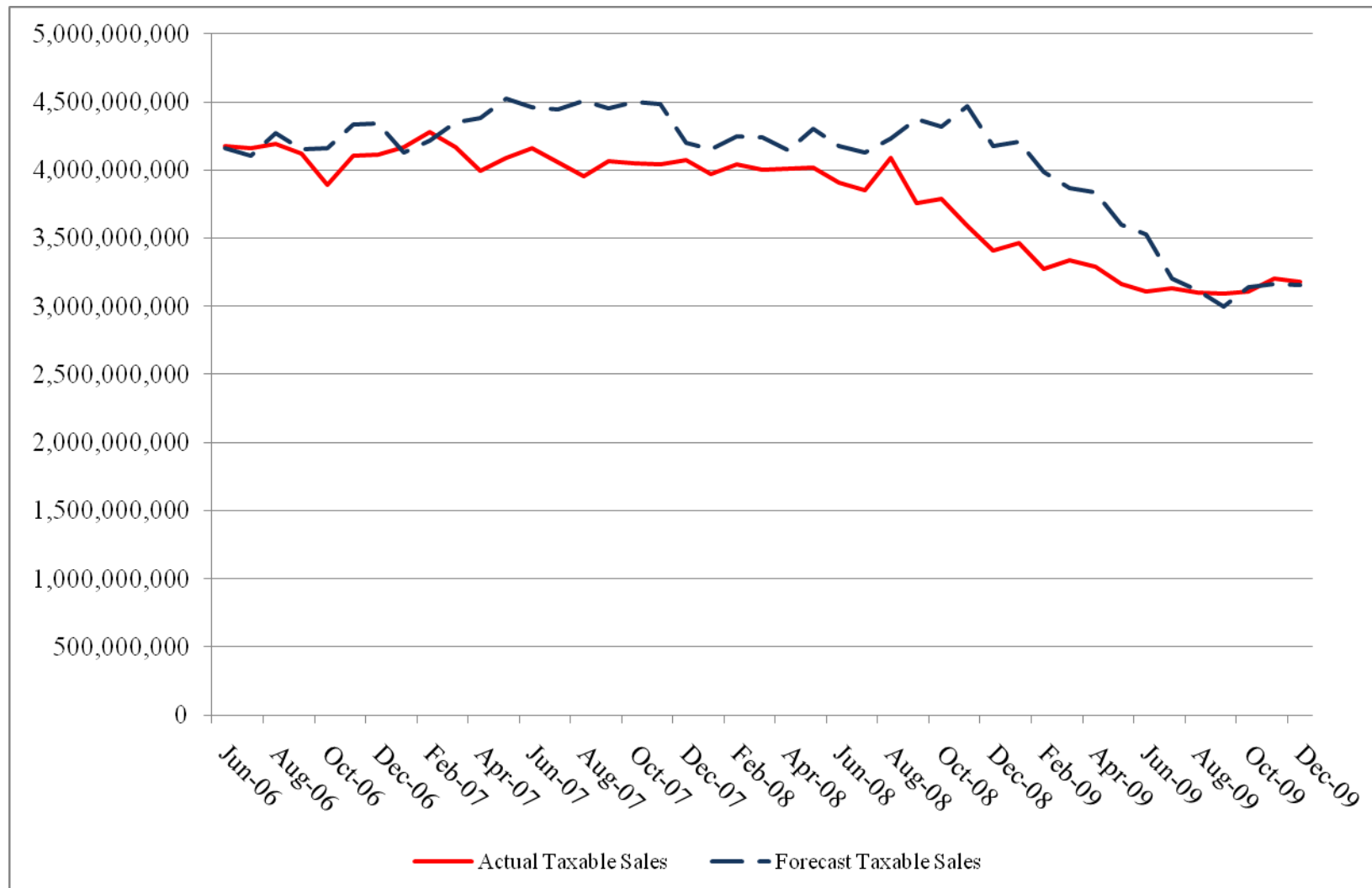


**Figure 6: Forecast and Actual Gross Gaming Revenue: Random-Walk Model**

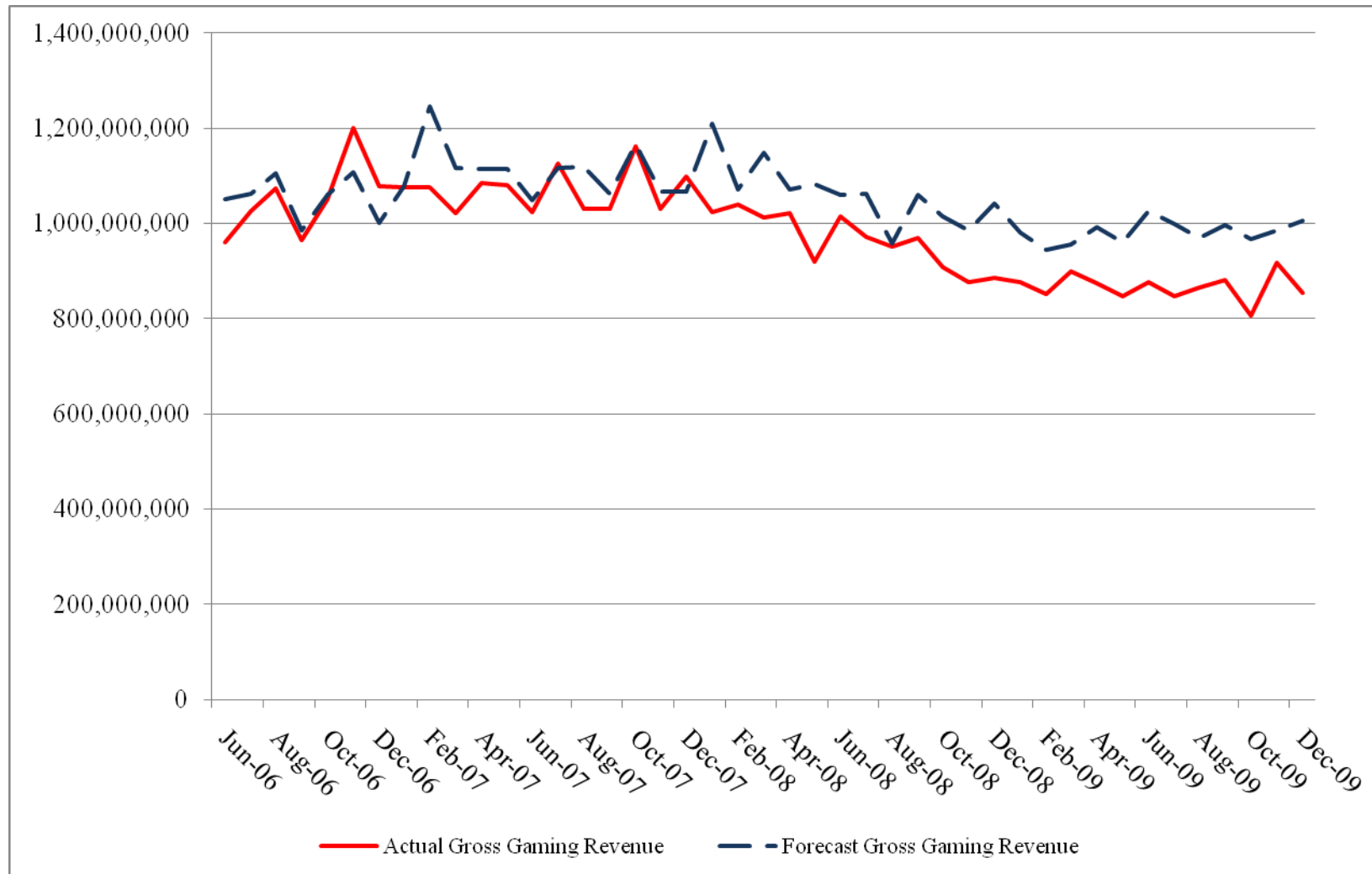




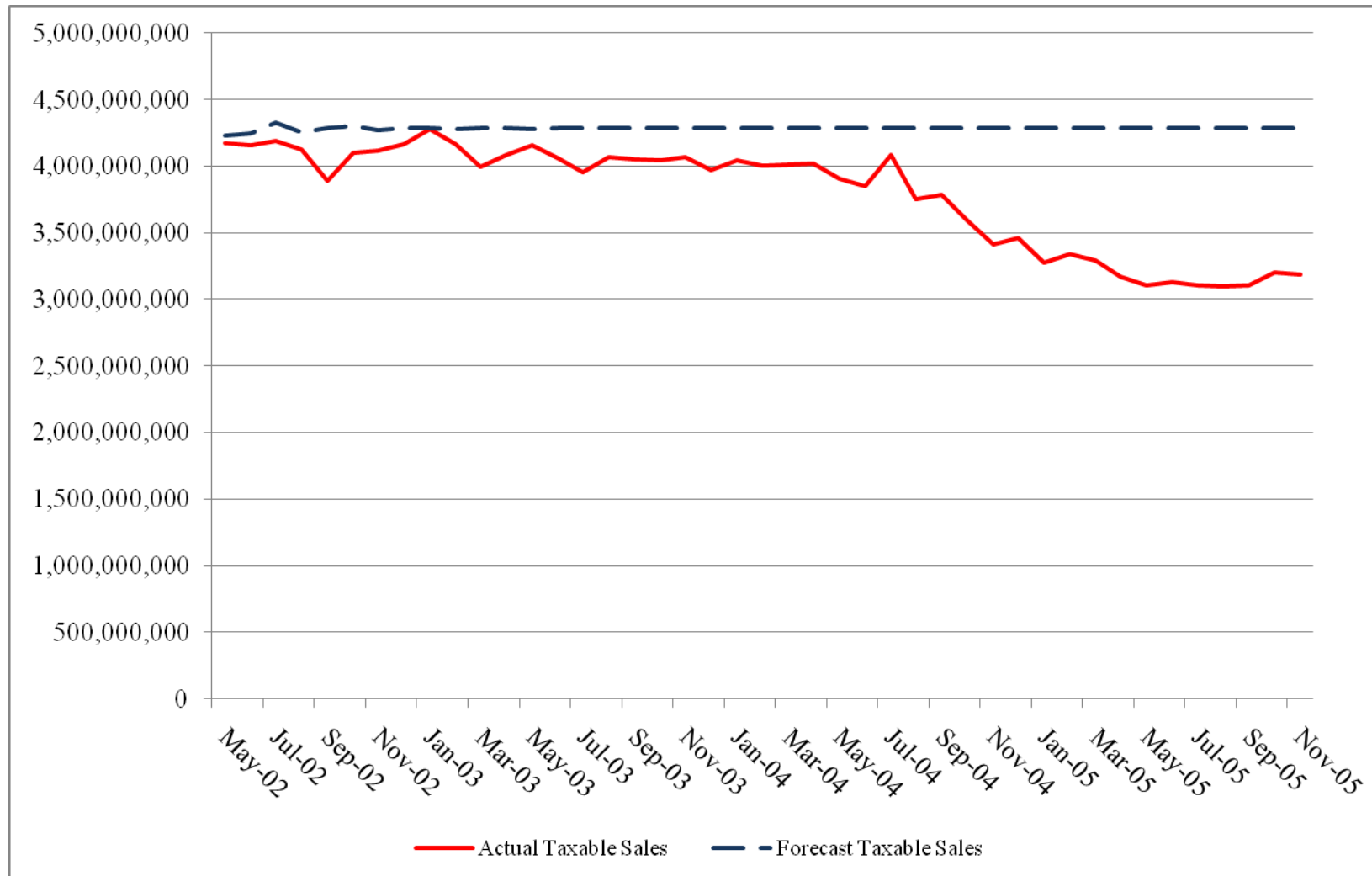
**Figure 7: Forecast and Actual Taxable Sales: Optimal Semi-Parametric Model**



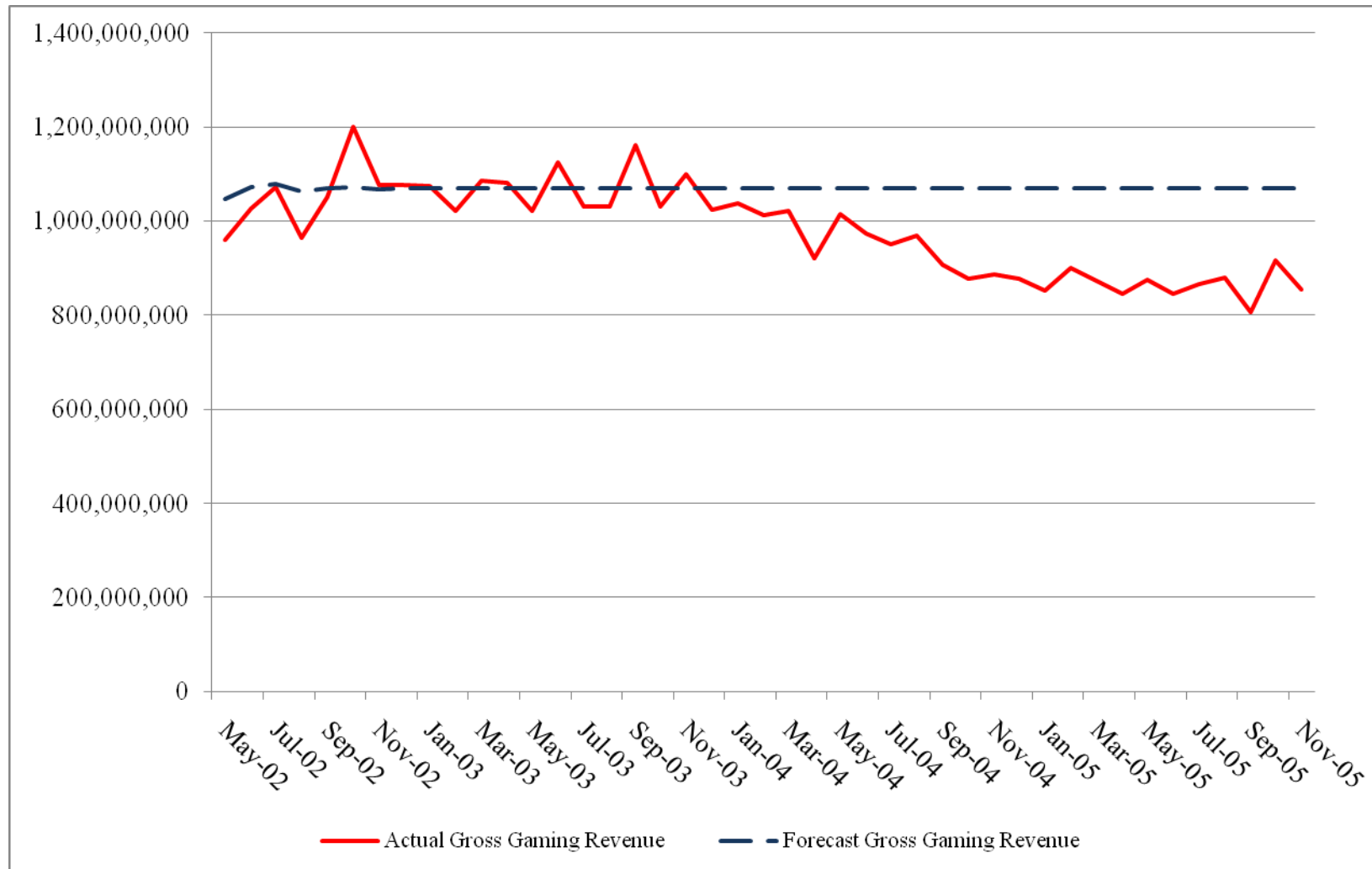
**Figure 8: Forecast and Actual Gross Gaming Revenue: Optimal Semi-Parametric Model**



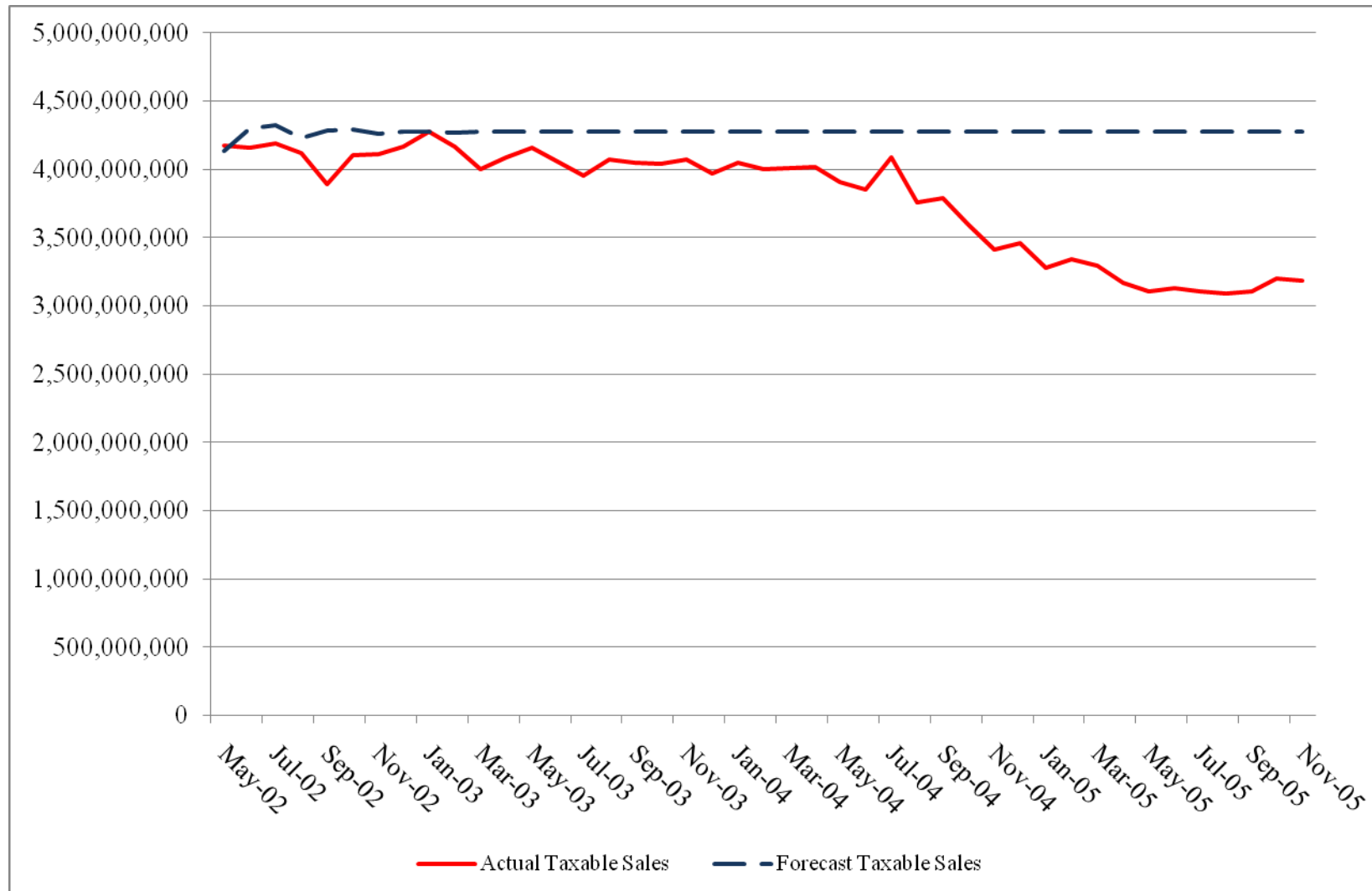
**Figure 9: Forecast and Actual Taxable Sales: Smooth Transition Autoregressive Model**



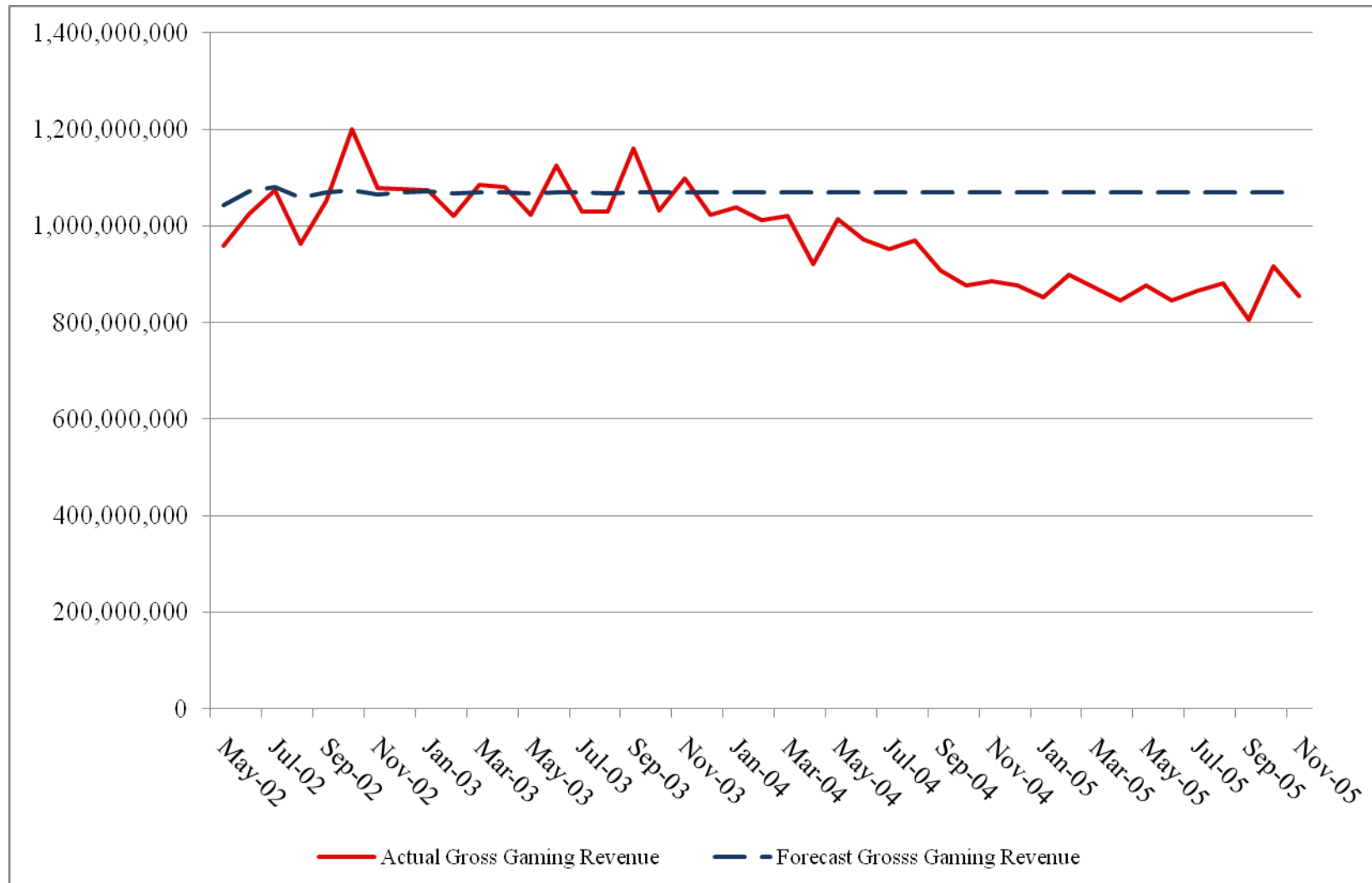
**Figure 10: Forecast and Actual Gross Gaming Revenue: Smooth Transition Autoregressive Model**



**Figure 11: Forecast and Actual Taxable Sales: Artificial Neural Network Autoregressive Model**



**Figure 12: Forecast and Actual Gross Gaming Revenue: Artificial Neural Network Autoregressive Model**



## Appendix:

For each of the one- to twenty-four-months-ahead forecasts, we test whether the gain (loss) in the RMSE from the AR-ANN model relative to the five other alternative models (random walk, VAR, VECM, SP, STAR) proves significant, using the Diebold and Mariano [DM] (1995) across model forecast comparison test. The relevant test-statistic is as follows:

$$(A1) \quad \hat{dp} = P^{-1/2} \frac{\sum_{t=R-h+1}^{T-1} (f(\hat{v}_{0,t+h}) - f(\hat{v}_{1,t+h}))}{\hat{\sigma}_p},$$

where  $R$  and  $P$ , respectively, denote the estimation and the prediction periods,  $f$  denotes some generic loss function, which we define as quadratic (i.e.,  $f[v_{0,t+h}] = v_{0,t+h}^2$ ),  $h \geq 1$  equals the forecast horizon,  $\hat{v}_{0,t+h}$  and  $\hat{v}_{1,t+h}$  are  $h$ -step-ahead prediction errors for models 0 and 1, where 0 stands for the AR-ANN model and 1 stands, in turn, for the other five models. We construct the statistic using Newey and West (1987) consistent estimators. Finally, we define  $\hat{\sigma}_p^2$  as follows:

$$(A2) \quad \begin{aligned} \hat{\sigma}_p^2 = & \frac{1}{P} \sum_{t=R-h+1}^{T-1} \left( f(\hat{v}_{0,t+h}) - f(\hat{v}_{1,t+h}) \right)^2 \\ & + \frac{2}{P} \sum_{j=1}^{lp} w_j \sum_{t=R-h+1+j}^{T-1} \left( f(\hat{v}_{0,t+h}) - f(\hat{v}_{1,t+h}) \right) \left( f(\hat{v}_{0,t+h-j}) - f(\hat{v}_{1,t+h-j}) \right), \end{aligned}$$

where  $w_j = 1 - \frac{j}{lp+1}$ ,  $lp = o(P^{1/4})$ . The hypotheses that we test are as follows:

$$H_0 : E(f(v_{0,t+h}) - f(v_{1,t+h})) = 0, \text{ and}$$

$$H_A : E(f(v_{0,t+h}) - f(v_{1,t+h})) \neq 0.$$

The DM test-statistic converges to a standard normal distribution under the null hypothesis.